Chapter 6

Work, Energy and Power

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Introduction

Do you often ask, "why should I study Physics"? What has this got to do with my life? Well, you might be the first person to invent a car that runs on waste or nuclear fuel or a space ship that is faster than speed of light ... or for that matter a person who comes out with a solution to technical problems. You can reason from basic concepts all the way to valid conclusions about the reality around you, and this understanding gives the fun. Physics is also the foundation of all engineering and technology. Could any engineer design a space ship, an LED TV or a mobile phone which all of you use without first understanding the basic laws of physics? It is one of the fundamental goals of physics to understand the terms 'work', 'energy' and 'power'. The topics are indeed important. Success of a civilization depends on acquiring and efficiently 'using' energy. We talk about work done by a machine and its power. In physics, these terms have different meaning. A little challenge for the inquisitive brains. Assuming that a racing car does not lose traction, the time it takes to race from rest through a distance s depends mainly on engine's power P. Derive the time t in terms of s and P assuming that the power is constant. You will be able to answer this after studying this chapter. Answer to this is given in one of the solved examples. You may compare your answer with our solution. Our aim is to understand these three physical quantities 'work', 'energy' and 'power'. But we need to understand a mathematical concept, namely the scalar product of two vectors before proceeding to understand this chapter. In this chapter, we shall study notion of work and kinetic energy, the work-energy theorem, work, kinetic energy, work done by a variable force, the work-energy theorem for a variable force, the concept of potential energy, the conservation of mechanical energy, motion in a vertical circle, the potential energy of a spring, various forms of energy, the law of conservation of energy, power, collisions (elastic and inelastic in one and two dimensions).

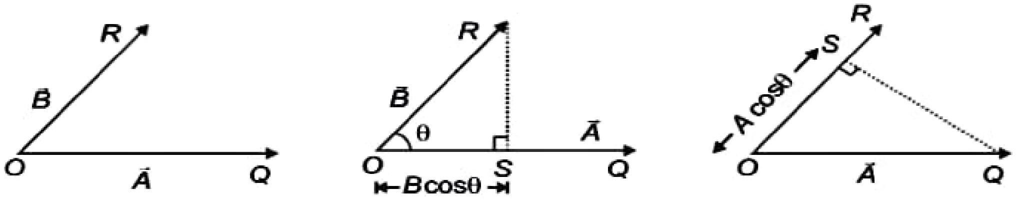
The Scalar Product

There are two ways of multiplying vectors (i) scalar product (ii) vector product. We shall study the latter in next chapter. The **scalar product** or **dot product** of any two vectors $\vec{A}$ and $\vec{B}$, denoted as $\vec{A}\cdot\vec{B}$ (read as $\vec{A}$ dot $\vec{B}$) is defined as

$$\vec{A}\cdot\vec{B} = AB\cos\theta \qquad ...(1)$$

where A and B are magnitudes of vectors $\vec{A}$ and $\vec{B}$ respectively and θ is the smaller angle between them. Dot product is called scalar product as A, B and $\cos\theta$ are scalars. Both vectors have a direction but their scalar product does not have a direction.

Geometrical interpretation:



(a) (b) (c)

Fig. : 1

Let $\vec{A}$ and $\vec{B}$ be represented by $\overrightarrow{OQ}$ and $\overrightarrow{OR}$ respectively. Let $\angle QOR$ be θ. In figure (b) draw $RS \perp OQ$.

$\therefore$ $OS = B\cos\theta$ is the projection of $\vec{B}$ onto $\vec{A}$. In figure (c), draw $QS \perp OR$

$\therefore$ $OS = A\cos\theta$ is the projection of $\vec{A}$ onto $\vec{B}$.

$$\vec{A}\cdot\vec{B} = AB\cos\theta = A(B\cos\theta) = B(A\cos\theta)$$

$\therefore$ $\vec{A}\cdot\vec{B}$ is the product of magnitude of $\vec{A}$ and component of $\vec{B}$ along $\vec{A}$ (or projection of $\vec{B}$ onto $\vec{A}$). Alternatively, it is the product of magnitude of $\vec{B}$ and the component of $\vec{A}$ along $\vec{B}$ (or projection of $\vec{A}$ onto $\vec{B}$).

Properties :

(i) Dot product is commutative

i.e., $\boxed{\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}}$ ...(2)

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$$\vec{B}\cdot\vec{A} = BA\cos\theta$$

$\therefore$ $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$ as $AB\cos\theta = BA\cos\theta$

(*i.e.*, dot product is commutative)

(ii) Dot product is distributive

i.e., $\boxed{\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}}$ ...(3)

(iii) Dot product of a vector with itself gives square of its magnitude

i.e., $\boxed{\vec{A}\cdot\vec{A} = AA\cos 0 = A^2}$ ...(4)

(iv) $\boxed{\vec{A}\cdot(\lambda\vec{B}) = \lambda(\vec{A}\cdot\vec{B})}$ ...(5)

where λ is a real number.

(v) $\boxed{\lambda\vec{A}\cdot(\lambda\vec{B}) = \lambda^2(\vec{A}\cdot\vec{B})}$ ...(6)

where λ is a real number.

Special cases :

Case-I : Two vectors are perpendicular (*i.e.*, $\theta = 90°$ also called orthogonal vectors)

$$\vec{A}\cdot\vec{B} = AB\cos 90° = 0$$

For unit vectors $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

Case-II : Two vectors are parallel (*i.e.*, $\theta = 0°$)

$$\vec{A}\cdot\vec{B} = AB\cos 0 = AB(1) = AB$$

For unit vectors

$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$$

Case-III : Two vectors are antiparallel (*i.e.*, $\theta = 180°$)

$$\vec{A}\cdot\vec{B} = AB\cos 180 = -AB, \text{ As } \cos 180 = -1$$

Understanding dot product in cartesian coordinates :

Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ & $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\begin{aligned}
\vec{A}\cdot\vec{B} &= (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) \\
&= A_x\hat{i}\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) + A_y\hat{j}\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) + A_z\hat{k}\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) \\
&= A_xB_x(\hat{i}\cdot\hat{i}) + A_xB_y(\hat{i}\cdot\hat{j}) + A_xB_z(\hat{i}\cdot\hat{k}) + A_yB_x(\hat{j}\cdot\hat{i}) + A_yB_y(\hat{j}\cdot\hat{j}) + A_yB_z(\hat{j}\cdot\hat{k}) + A_zB_x(\hat{k}\cdot\hat{i}) + A_zB_y(\hat{k}\cdot\hat{j}) \\
&\quad + A_zB_z(\hat{k}\cdot\hat{k}) \\
&= A_xB_x(1) + A_xB_y(0) + A_xB_z(0) + A_yB_x(0) + A_yB_y(1) + A_yB_z(0) + A_zB_x(0) + A_zB_y(0) + A_zB_z(1)]
\end{aligned}$$

$(\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1 \text{ and } \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0)$

$= A_xB_x + A_yB_y + A_zB_z$

$$\boxed{\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z} \qquad \ldots(7)$$

$$\Rightarrow \quad \vec{A}\cdot\vec{A} = A_xA_x + A_yA_y + A_zA_z$$

$$\text{or} \quad A^2 = A_x^2 + A_y^2 + A_z^2 \qquad \ldots(8)$$

As $\vec{A}\cdot\vec{A} = (A)(A)\cos 0 = A^2$

Example 1 : *If the magnitude of two vectors are 4 and 6 and the magnitude of their scalar product is* $12\sqrt{2}$*, what is the angle between the vectors?*

Solution : $A = 4, B = 6, \vec{A}\cdot\vec{B} = 12\sqrt{2}$

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{12\sqrt{2}}{4\times 6} = \frac{1}{\sqrt{2}}$$

$$\cos\theta = \cos 45°$$

$$\therefore \quad \boxed{\theta = 45°}$$

Example 2 : *Find the angle between force $\vec{F} = (5\hat{i} + 4\hat{j} + 5\hat{k})$ unit and displacement $\vec{d} = (3\hat{i} + 4\hat{j} - 3\hat{k})$ unit. Also find the projection of $\vec{F}$ on $\vec{d}$.*

Solution : $\vec{F}\cdot\vec{d} = F_x d_x + F_y d_y + F_z d_z = 5(3) + 4(4) + 5(-3) = 16$ unit

$\vec{F}\cdot\vec{F} = F^2 = F_x^2 + F_y^2 + F_z^2 = 5^2 + 4^2 + 5^2 = 66$ unit

$\vec{d}\cdot\vec{d} = d^2 = d_x^2 + d_y^2 + d_z^2 = 3^2 + 4^2 + (-3)^2 = 34$ unit

$\vec{F}\cdot\vec{d} = Fd\cos\theta$

$$\therefore \quad \cos\theta = \frac{\vec{F}\cdot\vec{d}}{Fd} = \frac{16}{\sqrt{66}\sqrt{34}} = \frac{16}{47.37} = 0.34$$

$\therefore \quad \theta = \cos^{-1}(0.34)$

Projection of $\vec{F}$ on $\vec{d} = F\cos\theta = \sqrt{66} \times 0.34 = 2.76$

Try Yourself

1. If the magnitudes of two vectors are 3 and 4 and magnitude of their scalar product is 6, what is the angle between the vectors?

   Hint : $A = 3, B = 4 \qquad \vec{A}\cdot\vec{B} = 6$

   $$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$$

2. If the magnitudes of two vectors are 2 and 3, and magnitude of their scalar product is $2\sqrt{3}$, what is the angle between the vectors?

   Hint : $A = 2,\ B = 3 \qquad \vec{A}\cdot\vec{B} = 2\sqrt{3}$

   $$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$$

3. Find the angle between the force $\vec{F} = (5\hat{i} + 4\hat{j} - 3\hat{k})$ units and displacement $\vec{d} = (3\hat{i} + 4\hat{j} + 5\hat{k})$ unit. Also, find the projection of $\vec{F}$ on $\vec{d}$.

   Hint : $\vec{F}\cdot\vec{d} = F_x d_x + F_y d_y + F_z d_z$

   $\vec{F}\cdot\vec{F} = F^2 = F_x^2 + F_y^2 + F_z^2 \qquad \vec{d}\cdot\vec{d} = d^2 = d_x^2 + d_y^2 + d_z^2$

   $$\cos\theta = \frac{\vec{F}\cdot\vec{d}}{Fd}$$

   Projection of $\vec{F}$ on $\vec{d}$ = $F\cos\theta$

4. Find the angle between the $\vec{F} = (4\hat{i} + 3\hat{j})$ units and displacement $\vec{d} = (4\hat{j} - 3\hat{k})$ unit. Also find the projection of $\vec{F}$ on $\vec{d}$.

   Hint : $\vec{F}\cdot\vec{d} = F_x d_x + F_y d_y + F_z d_z$

   $\vec{F}\cdot\vec{F} = F^2 = F_x^2 + F_y^2 + F_z^2 \qquad \vec{d}\cdot\vec{d} = d^2 = d_x^2 + d_y^2 + d_z^2$

   $\cos\theta = \dfrac{\vec{F}\cdot\vec{d}}{Fd} \qquad$ Projection of $\vec{F}$ on $\vec{d} = F\cos\theta$

WORK

If a force $\vec{F}$ is applied on a body in a direction different from the direction of displacement of the body and the body undergoes a displacement $\vec{d}$ in the positive X-direction as shown in figure 2.

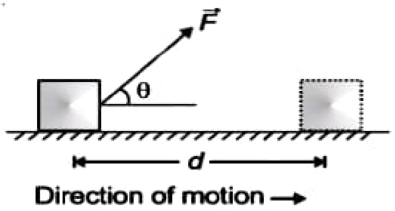


Fig. : 2

You may imagine a boy pulling a toy car with string

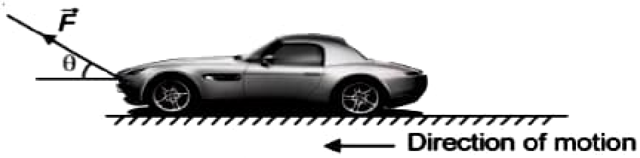


Fig. : 3

The work done by the force is defined as **the product of component of the force in the direction of the displacement and the magnitude of this displacement.**

So, $W = (F\cos\theta)d = \vec{F}\cdot\vec{d}$ ...(9)

Try to understand the logic behind this – body is moving horizontally so, we need to take product of effective component of force along the direction of motion (*i.e.*, $F\cos\theta$) with the displacement to get work done (*i.e.*, $F(\cos\theta)d$ or $\vec{F}\cdot\vec{d}$). Force $\vec{F}$ and displacement $\vec{d}$ are vectors but work (W) is scalar as dot product is a scalar ($(F\cos\theta)d$). No direction is associated with it. If force is applied along the direction of motion or body moves along the direction of force *i.e.*, $\theta = 0$.

$W = (F\cos 0)d = Fd$ As $\cos 0° = 1$ (This is what you studied in IX class).

In terms of rectangular components, $\vec{F}$ and $\vec{d}$, are

$\vec{F} = F_x\hat{i} + F_y\hat{j} = F_z\hat{k};\ \vec{d} = x\hat{i} + y\hat{j} + z\hat{k}$

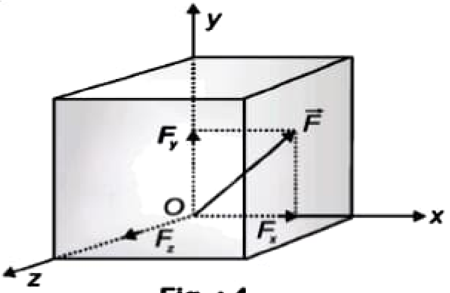


Fig. : 4

Imagine a point in the corner of a room and call it origin O. Now, imagine a point in space. Its distance from x-axis, y-axis and z-axis (length, breadth and height of room) are F_x, F_y and F_z respectively. Visualize a force as a vector in three dimensions. F_x, F_y and F_z are components of the force along x-axis, y-axis and z-axis respectively. Now imagine that the body moves under the action of this force and now imagine a point in space as the head of position vector $\vec{d}$ (final position of the body).

$\vec{d} = d_x\hat{i} + d_y\hat{j} + d_z\hat{k}$

where d_x, d_y and d_z are components of displacement vector $\vec{d}$ along *x*-axis, *y*-axis and *z*-axis respectively. *O* is tail of position vector $\vec{d}$ (starting position of the body taken as origin for simplicity). *A* is head of position vector $\vec{d}$ (final position of the body). Imagine that the force $\vec{F}$ applied in space along the direction of $\vec{F}$ (in figure 4) moves the body along the direction of $\vec{d}$ shown in figure 5.

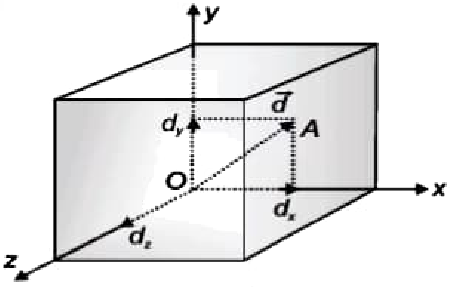


Fig. : 5

From Eq. (9), $W = \vec{F}\cdot\vec{d} = (F_x\hat{i} + F_y\hat{j} + F_z\hat{k})\cdot(d_x\hat{i} + d_y\hat{j} + d_z\hat{k})$

$$\boxed{W = F_x d_x + F_y d_y + F_z d_z} \qquad ...(10)$$

From equations (9) & (10) it is obvious that work is a scalar quantity *i.e.*, it has magnitude only and no direction. However it can be positive, negative or zero.

Positive work :

$W = \vec{F}\cdot\vec{d} = Fd\cos\theta$

So, work done is positive when θ is acute ($\theta < 90°$) as $\cos\theta$ is positive. (As you can see from the graph)

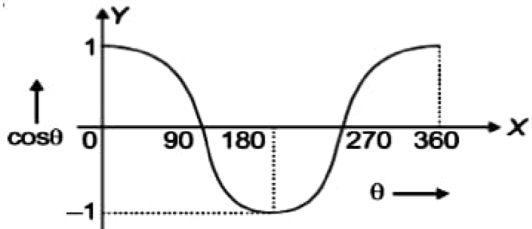


Fig. : 6

For example,

(i) A spring is stretched, work done by the stretching force is positive (force and displacement are in same direction $W = Fd$)

(ii) A body falls freely under the action of gravity $\theta = 0°$, $\cos\theta = +1$. Work done by gravity on a body falling freely is positive. (Here, force of gravity and displacement are in the same direction).

(iii)

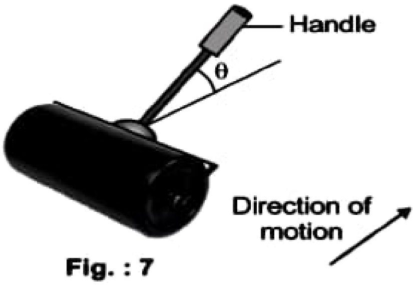


Fig. : 7

A lawn roller is pulled by a force along the handle at an acute angle, work done by the force is positive.

Negative work :

$W = \vec{F} \cdot \vec{d} = Fd\cos\theta$ ·

Work done is negative when θ is obtuse ($\theta > 90°$) $\cos\theta$ is negative. (Refer to figure 6, $\cos\theta$ is negative for $\theta > 90°$)

For example,

(i) Work done by the braking force on a moving vehicle is negative. Force is in a direction opposite to the direction of motion. $\theta = 180°$, $\cos 180 = -1$, $W = -Fd$ (Refer to figure 6)

Note : Work done by the applied force is not negative.

(ii) Work done by gravity on a body moving upwards is negative as angle between gravitational force $\vec{F}$ and displacement $\vec{d}$ is 180°. $\cos\theta = \cos 180° = -1$, so $W = -Fd$

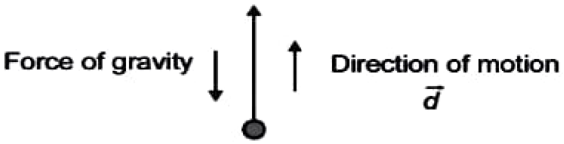


Fig. : 8

(iii) Imagine two equal charges $-Q$. Imagine moving charge B closer to charge A. Work done by electrostatic force of repulsion between them is negative.

Note : Positive work on an arbitrary system means a transfer of energy to the system.
Negative work on an arbitrary system means a transfer of energy from the system.

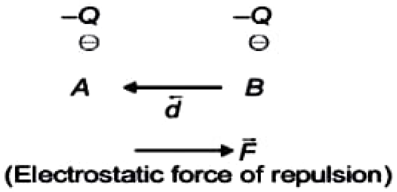


Fig. : 9

Zero Work:

$W = \vec{F} \cdot \vec{d} = Fd\cos\theta = 0$ when $F = 0$ or $d = 0$ or $\cos\theta = 0$ ($\theta = 90°$)

Work is zero when

(i) The force is zero. For example, A body moving on a smooth horizontal surface is not acted upon by a horizontal force (as there is no friction), but may undergo displacement. So, $W = 0$ as $F = 0$ even though $d \neq 0$ and $\cos\theta \neq 0$.

(ii) The displacement is zero. For example, when you push hard against a train to move it. Force exerted by you does no work. You have applied muscular force, internal energy has been used up, you get tired but no work has been done in the language of physics $W = 0$ as $d = 0$ even though $F \neq 0$ and $\cos\theta \neq 0$. Another example is that you study for six hours but no work has been done as $d = 0$. The work you have done is mental work, so you see that meaning of work is different from its usage in day-to-day life.

(iii) The force and displacement are mutually perpendicular *i.e.*,

$$\theta = \frac{\pi}{2} (= 90°), \cos\theta = 0° \text{ (Refer figure 6).}$$

For example, sun's gravitational force on the earth does no work (assuming the earth's orbit around the sun as perfectly circular). Earth's instantaneous displacement is tangential while the sun's gravitational force (centripetal force) is directed towards the centre.

$$\theta = \frac{\pi}{2}, \; W = Fd\cos\theta = 0.$$

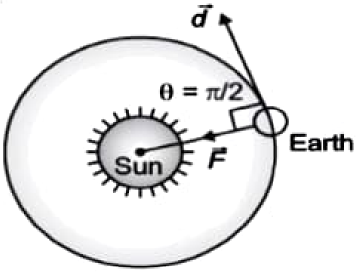


Fig. : 10

A body tied to one end of a string is rotated uniformly in a circle. Work done by the centripetal force applied along the string is zero as $\theta = 90°$, $W = Fd\cos\theta = Fd\cos 90° = 0$

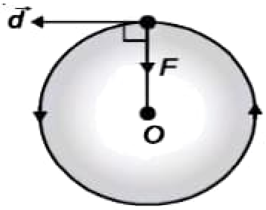


Fig. : 11

A coolie carrying a load on his head moves on a horizontal ground. Work done by him against gravitational force is zero as $\theta = 90°$. Gravitational force is acting vertically downward and displacement is in horizontal direction, thus $\theta = 90°$. Work done by tension in the string of a simple pendulum is again zero as tension is always perpendicular to the displacement.

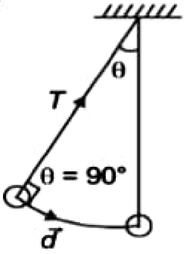


Fig. : 12

Note : *We have assumed ideal conditions i.e., there is neither any resistance due to air nor any opposing force of friction. And in the case of circular motion if the motion is* ***not uniform****, a tangential force acts on the body in addition to radial force. Work done by radial force = 0 but work done by tangential force ≠ 0 i.e., kinetic energy of the body will change.*

Dimensions and Units of Work :

Taking $\theta = 0°$

Work = force × distance

$W = [MLT^{-2}]\,[L]$

$\boxed{W = [ML^2T^{-2}]}$

This is the dimensional formula for work.

There are two types of units of work:

1. **Absolute Units**

(i) **SI units** is joule (absolute unit of work in SI)

Work is said to be one joule, when a force of one newton actually moves a body through a distance of one metre in the direction of applied force.

$W = Fd\cos\theta$

$= 1 \text{ newton} \times 1 \text{ metre} \times \cos 0°$

$= 1$ N-m

$= 1$ joule

(ii) **CGS units** is Erg (absolute unit of work in CGS)

Work is said to be one erg, when a force of one dyne actually moves a body through a distance of one cm in the direction of applied force.

$W = Fd\cos\theta$

$= 1 \text{ dyne} \times 1 \text{ cm} \times \cos 0$

$= 1$ dyne cm

$= 1$ erg

Relation between joule and erg

$1 \text{ joule} = 1 \text{ N} \times 1 \text{ m}$

$= 10^5 \text{ dyne} \times 10^2 \text{ cm as } 1 \text{ N}$

$= 10^5$ dyne

$= 10^7$ dyne cm

$\boxed{1 \text{ joule} = 10^7 \text{ erg}}$

2. **Gravitational Unit**

(i) **SI unit** is kilogram-metre (kg-m). It is the gravitational unit of work in SI like joule is the absolute unit of work in SI. **Work done is said to be one kg-m, when a force of 1 kgf (or 1kg wt) moves a body through a distance of 1 m in the direction of the applied force.**

$W = Fd\cos\theta$

$1 \text{ kg-m} = 1 \text{ kgf} \times 1 \text{ m} \times \cos 0$

$= 9.8 \text{ N} \times 1 \text{ m}$

$= 9.8$ joule (as $1 \text{ kgf} = 9.8$ N)

$\boxed{1 \text{ kg-m} = 9.8 \text{ J}}$

(ii) **CGS unit** is gram-centimetre (*g*-cm) (Gravitational unit of work is CGS system). **Work done is said to be one g-cm, when a force of 1 gf moves a body through a distance of 1 cm in the direction of the applied force.**

$W = Fd\cos\theta$

$1 \text{ g-cm} = 1 \text{ gf} \times 1 \text{ cm} \times \cos 0°$

$= 980 \text{ dyne} \times 1 \text{ cm} \times 1$ as 1 gf

$= 980 \text{ dyne}$ $\boxed{1 \text{ g.cm} = 980 \text{ erg}}$

Relation between kg-m and g-cm

$1 \text{ kg.m} = 10^3 \text{g} \times 10^2 \text{ cm}$

$\boxed{1 \text{ kg.m} = 10^5 \text{ g-cm}}$

Alternative units of work/energy in Joule

Table-1

Electron volt	1.6×10^{-19} J
Calorie (cal)	4.186 J
Kilowatt hour (kWh)	3.6×10^6 J

Example 3 : *A gas filled in a cylinder fitted with a movable piston is allowed to expand. What is the nature of the work done by the gas?*

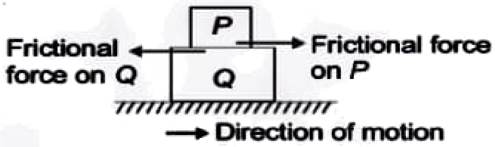
Solution : Positive, as force due to gaseous pressure and displacement of piston are in the same direction.

Try Yourself

5. A body *P* is on a body *Q* and moving with it. Comment on the nature of work done by friction on body *P* and on body *Q* at their contact surface.

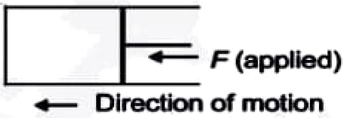
Hint : Force of friction on body *P* is in the direction of displacement

while force of friction on body *Q* is in a direction opposite to displacement



6. A gas filled in a cylinder fitted with a movable piston is compressed. What is the nature of work done by the applied force?

Hint :



Example 4 : *A body is pushed through 5 m across a surface offering 75 N resistance. How much work is done by the (i) applied force (ii) resisting force?*

Solution : (i) $F = 75$ N, $d = 5$ m

Work done by applied force $= W = Fd\cos 0°$

$= 75 \times 5 \times 1 = 375$ J

(ii) Work done by resisting force $= Fd\cos 180°$

$= 75 \times 5 \times (-1) = -375$ J

Try Yourself

7. A car comes to a sliding stop in 5 m. During this process, the force on the car due to road is 100 N and is directed opposite to the motion.

   (a) How much work does the road do on the car?

   (b) How much work does the car do on the road?

   Hint : (a) The stopping force and displacement make an angle of 180° with each other

   $W_r = Fd\cos\theta$

   $F = 100$ N, $d = 5$ m, $\cos 180° = -1$

   (b) An equal and opposite force acts on the road due to the car but road undergoes no displacement.

8. A 100 kg block is kept at the back of a truck moving with an acceleration of 2 ms^{-2}. There is no relative motion. What is the work done by frictional force on the block as truck is displaced by 2 m w.r.t., Ground

   Hint : Frictional force, $f = ma$

   $w_1 = fd\cos\theta$ $d = 2$ m

KINETIC ENERGY

As discussed earlier, kinetic energy K of an object of mass m is given as

$$\boxed{k = \frac{1}{2}m\vec{v}\cdot\vec{v} = \frac{1}{2}mv^2} \quad \ldots(11)$$

It is a scalar quantity. It is a measure of the work an object can do because of its motion. Sailing ships use KE of the wind. KE of a fast-flowing stream has been used for grinding corn.

Table-2 : KE for various objects

Object	Mass (kg)	Speed (ms^{-1})	k(J)
Car	2000	25	6.3×10^5
Running athlete	70	10	3.5×10^3
Bullet	5×10^{-2}	200	10^3
Stone dropped from 10 m	1	14	10^2
Rain drop at terminal speed	3.5×10^{-5}	9	1.4×10^{-3}
Air molecule	$\simeq 10^{-26}$	500	$\simeq 10^{-21}$

Alternative method for expression for KE (Calculus method):

Let mass of the body be m, which is initially at rest ($u = 0$). Let force applied be $\vec{F}$ causing a small displacement $d\vec{s}$ in the direction of the applied force. Small amount of work, dW done by the force is

$$dW = \vec{F}\cdot d\vec{s}$$

$$dW = F\,ds\cos 0^\circ$$

$$dW = Fds \qquad \ldots(12)$$

$$F = ma = m\frac{dv}{dt}$$

From equation (12), $dW = \left(m\frac{dv}{dt}\right)ds$

$$dW = m\left(\frac{ds}{dt}\right)dv$$

$$dW = mvdv \left(\text{as } v = \frac{ds}{dt}\right)$$

Total work done by the force in increasing the velocity from zero to v is

$$W = \int_0^v mvdv = m\int_0^v vdv = m\left[\frac{v^2}{2}\right]_0^v$$

$$W = \frac{1}{2}mv^2$$

$$\boxed{\text{KE of the body} = W = \frac{1}{2}mv^2} \qquad \text{KE} \propto m \text{ and KE} \propto v^2$$

Relation between KE and linear momentum

Let mass of body be m and velocity of body be v.

$p = mv$ = linear momentum

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2m}(m^2v^2)$$

$$\text{KE} = \frac{p^2}{2m} \qquad \text{as } p = mv$$

Note :

1. *KE depends upon the frame of reference e.g., KE of a person of mass m sitting in a car moving with velocity v is* $\frac{1}{2}mv^2$ *in earth's frame and KE of same person is zero in the frame of the car.*
2. *KE is always positive.*
3. KE = $\frac{1}{2}mv^2$ *holds even when magnitude or direction of force changes. So, expression is valid irrespective of how the body acquires the velocity v.*

Stopping Distance

(i) When a body slide a distance s on a rough horizontal surface with velocity v, its stopping distance

$$\boxed{s = \frac{v^2}{2\mu g}}$$, where μ is coefficient of friction.

or, $$\boxed{s = \frac{\frac{1}{2}mv^2}{\mu mg} = \frac{\text{Kinetic energy}}{\text{Retarding force}}}$$

(ii) Stopping distance and stopping time of a lorry and a car

$$\boxed{\frac{x_1}{x_2}=\frac{Fx_1}{Fx_2}=\frac{K_1}{K_2} \text{ and } \frac{t_1}{t_2}=\frac{Ft_1}{Ft_2}=\frac{p_1}{p_2}}$$, when retarding forces (F) are same.

Here K and p represent kinetic energy and momentum respectively.

Case-I :

When $K_1 = K_2$ (*i.e.*, kinetic energies are same)

$$x_1 = x_2 \text{ but } \frac{t_1}{t_2}=\frac{p_1}{p_2}=\frac{\sqrt{2m_1K_1}}{\sqrt{2m_2K_2}}=\sqrt{\frac{m_1}{m_2}}$$

i.e., $\boxed{\text{stopping distances are same but the car will stop in less time}}$

Case-II :

When both have same momentum *i.e.*, $p_1 = p_2$

$$\frac{x_1}{x_2}=\frac{K_1}{K_2}=\frac{\frac{p_1^2}{2m_1}}{\frac{p_2^2}{2m_2}}=\frac{m_2}{m_1}$$

$\Rightarrow$ $x_1 > x_2$ if $m_1 < m_2$

Hence $\boxed{\text{stopping distance of car is more than that of a lorry}}$

But $\frac{t_1}{t_2}=\frac{p_1}{p_2}=1$

$\Rightarrow$ $t_1 = t_2$

$\boxed{\text{Both will stop in same time}}$

(iii) When a gas expands by a volume ΔV at a pressure P, the work done by the gas $\Delta W = P.\Delta V$

Example 5 : *In CID serial (on TV), Abhijeet fires a bullet of mass 100 g with a speed of 100 ms^{-1} on a soft plywood of thickness 4 cm. The bullet emerges with 10% of its initial KE. Find the emergent speed of the bullet.*

Solution : Initial KE of the bullet = $\frac{1}{2}mu^2 = \frac{1}{2}\times\frac{100}{1000}\times 100\times 100 = 500$ J

Final KE of the bullet = $\frac{10}{100}\times 500 = 50$ J

Let emergent speed of the bullet be v, $\frac{1}{2}mv^2 = 50$

$$\Rightarrow \quad v=\sqrt{\frac{2\times 50}{\frac{100}{1000}}}=31.6\text{ ms}^{-1}$$

Try Yourself

9. A bullet of mass 200 g hits a block of thickness 6 cm with a speed of 100 ms^{-1}. It emerges with 10% of its initial KE. Find its emergent speed.

   Hint : K_i = initial KE = $\frac{1}{2}mu^2$

   $m = \frac{200}{1000}\text{kg}$

   $u = 100 \text{ ms}^{-1}$

   Final KE = $0.1\ K_i = \frac{1}{2}mv^2$

10. A body of mass 100 g moving on a test track has final KE of 50 J after travelling a distance of 10 cm. Assuming 90% loss of energy due to friction, find the initial speed of the body.

    Hint : $\frac{1}{2}mv^2 = 50\text{J}$

    If initial KE is K_i then $K_i \times \frac{10}{100} = 50$ Find K_i.

    $K_i = \frac{1}{2}mu^2$ $\qquad m = \frac{100}{1000}\text{kg}.$

    Find u.

WORK DONE BY A VARIABLE FORCE

In daily life, a constant force is rare *e.g.*, if we want to calculate work done by an engine of a car, force exerted by engine is a variable force (car accelerates, travels with constant speed for some time, brakes are applied ...it comes to rest ... again accelerates ...)

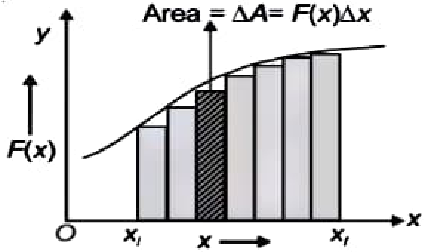


Fig. : 13

Fig. 13 shows plot of a varying force in one dimension. If displacement Δx is small, force $F(x)$ can be regarded as approximately constant and the work done $\Delta W = F(x)\Delta x$. This is shown in Fig. 13 ΔW is the work done corresponding to small displacement Δx over which force is regarded constant. If we add successive rectangular areas in Fig. 13.

Total work done, $W \equiv \sum_{x_i}^{x_f} F(x)dx$ ...(13)

Where the summation is from initial position x_i to the final position x_f.

As an example, if we are talking about $\sum_{x=1}^{x=4} F(x)\Delta x$

It is = $F_1\Delta x + F_2\Delta x + F_3\Delta x + F_4\Delta x$

If the displacements are allowed to approach zero, then the number of terms in the sum increases without limit, but the sum approaches a definite value equal to the area under the curve in Fig. 14

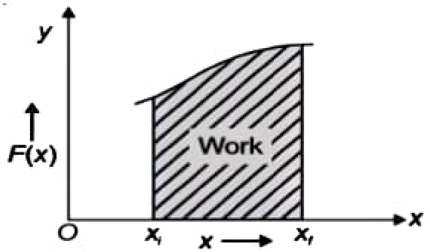


Fig. : 14

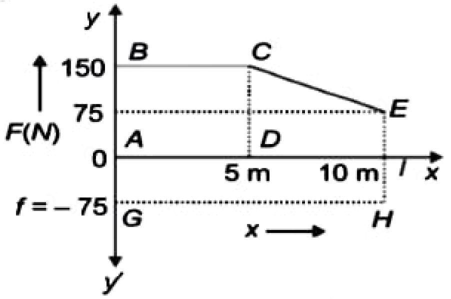
Adding the areas of all the rectangles we find that for $\Delta x \to 0$, the area under the curve is exactly equal to the work done by $F(x)$.

$$W = \lim_{\Delta x \to \infty} \sum_{x_i}^{x_f} F(x)\Delta x = \int_{x_f}^{x_f} F(x)dx \qquad \ldots(14)$$

Where 'lim' stands for the limit of the sum when Δx tends to zero. Thus, for a varying force the work done can be expressed as a definite integral of force over displacement.

Example 6 : *A gardener pushes a lawn roller on a rough surface. He applies a force of 150 N over a distance of 5 m. After that, he gets tired and his applied force reduces linearly with distance to 75 N. The total distance moved is 10 m. Plot the force applied by the gardener and the frictional force, which is 75 N versus displacement. Calculate the work done by the two forces over 10 m.*

Solution : Plot of the force $\vec{F}$ applied by the gardener and the opposing frictional force f vs displacement.



At x = 10 m, F = 75 N (≠ 0) frictional force, $f = |\vec{f}| = 75$ N. It opposes motion and acts in a direction opposite to $\vec{F}$. It is therefore shown on the negative side of the force axis.

The work done by the Gardener is

W_F = Area of rectangle *ABCD* + Area of trapezium *CEID*

$$= 150 \times 5 + \frac{1}{2} \times (150 + 75) \times 5 = 750 + 562.5 = 1312.5 \text{ J}$$

Work done by frictional force is

W_f = Area of rectangle *AGHI*

$= (-75) \times 10 = -750$ J

The area on the negative side of the force axis has a negative sign.

Work Done by Spring Force

Consider a force is applied on a spring having force constant k to stretch it from natural length to produce elongation x in it. The work done on spring is given by

$$W = \int_0^x F dx = \int_0^x kx\, dx$$

$$W = \frac{1}{2}kx^2$$

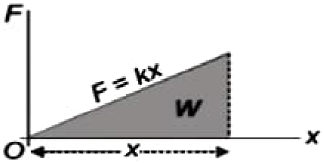
***Note :** The work done by the stretching or compressing force (i.e., the force applied on the spring by an external agent) is positive. But the work done by the spring force (–kx) is negative because the force exerted by the spring is always opposite to the elongation or contraction.*

The work done by the force applied by the spring on the agent is

$$W = \int_{x_1}^{x_2} F\, dx = \int_{x_1}^{x_2} (-kx)\, dx = -\frac{1}{2}k(x_2^2 - x_1^2)$$

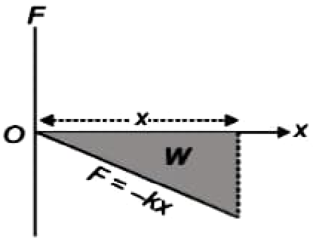
or $\boxed{W = -\frac{1}{2}kx^2}$, when $x_1 = 0$, $x_2 = \pm x$

(a)



The shaded area is equal to the positive work done by the stretching force on the spring

(b)



The shaded area is the negative work done by the spring force on the stretching agent

Try Yourself

11. A force $F = (5 + 2x)$ acts on a particle in x direction, where F is in newton and x is in metre. Find the work done by this force during a displacement from $x = 0$ to $x = 1$ m.

Hint : $F = (5 + 2x)$

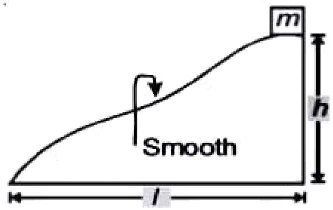
$$dW = \vec{F} \cdot d\vec{x} = (5+2x)dx$$

Total work done, $W = \int_{x=0}^{x=1} (5+2x)dx$

12. A particle moves along x-axis from $x = 0$ to $x = 2$ m under the influence of a force F (in N) given by $F = 3x^2 + 2x - 5$. Calculate the work done.

    Hint : $W = \int_0^2 (3x^2 + 2x - 5)dx$

13. A block of mass m is released along a smooth track shown in figure from its top point. Find the work done by gravity during its downward motion to the bottom of track.



14. If force $F = (2x + 3x^2)$ N is applied on a particle along x-axis, then find work done by it during motion of particle from $x = 0$ to $x = 2$ m.

NOTION OF WORK AND KINETIC ENERGY : THE WORK-ENERGY THEOREM

You are familiar with the following relation for rectilinear motion under constant acceleration a

$$v^2 - u^2 = 2as \qquad ...(15)$$

(u and v are the initial and final speeds and s is the distance covered)

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \left(\frac{1}{2}m\right)2as \qquad \text{(multiplying both sides of the equation (15) by } \frac{m}{2}\text{)}$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas = Fs \qquad ...(16) \qquad \text{(as } ma = F\text{, from Newton's II}^{\text{nd}}\text{ law)}$$

Let us generalise equation (15) to three dimensions by using vectors

$$v^2 - u^2 = 2\vec{a}\cdot\vec{d} \qquad ...(15a)$$

Multiplying both sides of equation (15a) by $\frac{m}{2}$, we get

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \left(\frac{1}{2}m\right)(2\vec{a}\cdot\vec{d})$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = m\vec{a}\cdot\vec{d} = \vec{F}\cdot\vec{d} \qquad ...(16a) \qquad \text{(as } m\vec{a} = \vec{F}\text{, from Newton's II}^{\text{nd}}\text{ law)}$$

This beautiful equation motivates us to give the definitions of work and energy. As you can see left side of the equation is the difference in the half the mass times the square of the speed from its initial value to its final value. Each of these quantities will be called the 'kinetic energy', denoted by K,

K_i = initial kinetic energy = $\frac{1}{2}mu^2$

K_f = final kinetic energy = $\frac{1}{2}mv^2$

The right side of the equation (16a) is the product of the component of the force along the displacement and the displacement. (Remember the scalar product). We call this 'work' and denote it by W *i.e.* $W = \vec{F}\cdot\vec{d}$

So, our equation (16a) becomes

$$\boxed{K_f - K_i = W} \qquad ...(17)$$

So, we have our beautiful work-energy (W-E) theorem : **The change in kinetic energy of a particle is equal to the work done on it by the net force**. Work refers to the force and displacement over which it acts. **Work is done by the net force over a specific displacement on the body.**

Example 7 : *A raindrop of mass 2 g falling from a height of 1.00 km, hits the ground with a speed of 40.0 ms^{-1}.*

(a) Find the work done by the gravitational force.

(b) Find the work done by the opposing resistive force (g = 10 ms^{-2})

Solution : (a) Assuming $u = 0$ and constant g = 10 ms^{-2}, the work done by the gravitational force is

$$W_g = mgh$$

$$= 2 \times 10^{-3} \times 10 \times 10^3 = \boxed{20\ \text{J}}$$

(b) $\Delta K = \frac{1}{2}mV^2 - 0 = \frac{1}{2} \times 2 \times 10^{-3} \times 40 \times 40 = 1.6\ \text{J}$

From the work-energy theorem

$\Delta k = W_g + W_r$ where W_r is the work done by the opposing resistive force.

$$1.6 = 20 + W_r$$

$$\boxed{W_r = -18.4\ \text{J}}$$

Try Yourself

15. A bullet of mass 10 g moving with a velocity of 300 ms^{-1} strikes a wooden block and comes out from other side with a velocity of 200 ms^{-1}. Find the work done by the resistive force on the bullet.

Hint : $\Delta K = K_f - K_i = W$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2;$$

$m = 10 \times 10^{-3}$ kg,

u = 300 m/s,

v = 200 m/s

16. A ball of mass 1 kg is projected with 10 ms^{-1} from the top of a tower of height 20 m. What is its speed when it is at vertical distance of 10 m below point of projection? (g = 10 ms^{-2})

Hint : $\Delta K = K_f - K_i = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = W_{\text{gravity force}}$

u = 10 m/s, v = ?,

m = 1 kg,

h = 10 m

THE WORK-ENERGY THEOREM FOR A VARIABLE FORCE

Confining to one dimension, rate of change of kinetic energy with time is

$$\frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right)$$

$$= \frac{1}{2}m\frac{d}{dt}(v^2) \text{ (as } m \text{ is constant)}$$

$$= \frac{1}{2}m\left(2v\frac{dv}{dt}\right)$$

$$= m\frac{dv}{dt}v$$

$$= Fv \qquad \left(\text{as } F = ma = \frac{mdv}{dt} \text{ (from Newton's II}^{nd} \text{ law)}\right)$$

$$\frac{dK}{dt} = F\frac{dx}{dt} \qquad \text{or} \qquad dK = Fdx$$

$$\int_{K_i}^{K_f} dK = \int_{x_i}^{x_f} Fdx$$

[Integrating from the initial position (x_i) to the final position (x_f)]

Where K_i and K_f are the initial and final kinetic energies corresponding to x_i and x_f respectively.

or $$\boxed{K_f - K_i = \int_{x_i}^{x_f} Fdx} \qquad \text{...(18a)}$$

Thus, $$\boxed{K_f - K_i = W} \qquad \text{...(18b) from (14)}$$

Thus, work-energy theorem is proved for a variable force.

Note :

(1) *W-E theorem is an integral form of Newtons IInd law. It does not, in general incorporate the complete dynamical information of IInd law, which is a relation between force and acceleration at any time. W-E theorem involves an integral over an interval of time. The temporal (time) information contained in IInd law is 'integrated over' and is not available explicitly.*

(2) *The W-E theorem is in scalar form whereas IInd law for two or three dimensions is in vector form. The information w.r.t. directions contained in Newton's IInd law is not present in it.*

Example 8 : *The position (x) of a particle of mass 1 kg moving along x-axis at time t is given by* $\left(x = \frac{1}{2}t^2\right)$ *metre. Find the work done by force acting on it in time interval from t = 0 to t = 3s.*

Solution : $x = \frac{1}{2}t^2 \Rightarrow v = \frac{dx}{dt} = \frac{1}{2}(2t) = t$

$\therefore$ At $t = 0$, $v_i = 0$

At $t = 3\text{s}$, $v_f = 3 \text{ ms}^{-1}$

According to W-E theorem,

$$W = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$= \frac{1}{2}\times 1\times 3^2 = 4.5 \text{ J}$$

EXERCISE

1. A particle moves from point $P(1, 2, 3)$ to $Q(2, 1, 4)$ under the action of a constant force $\vec{F} = (2\hat{i} + \hat{j} + \hat{k})$ N. Work done by the force is

(1) 2 J (2) 4 J

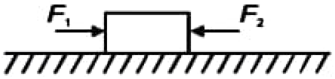
(3) 16 J (4) 8 J

2. Which of the following vector is perpendicular to the vector $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$?

(1) $\hat{i} + \hat{j} + \hat{k}$ (2) $4\hat{i} + 3\hat{j} - 2\hat{k}$

(3) $\hat{i} - 3\hat{j} + \hat{k}$ (4) $\hat{i} + 2\hat{j} - 2\hat{k}$

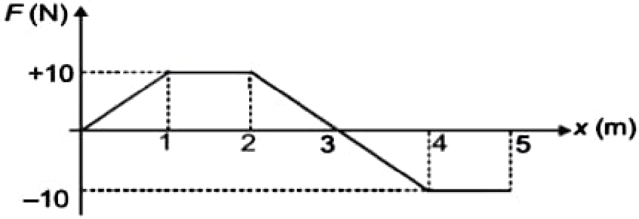
3. A block of mass 2 kg is placed on a smooth horizontal surface. Two forces F_1 = 20 N and F_2 = 5 N start acting on the block in opposite directions as shown. If block gets displaced by 5 m in the direction of net force then work done by F_2 is



(1) –75 J (2) 75 J

(3) –25 J (4) 25 J

4. A position dependent force F acting on a particle and its force-position curve is shown in the figure. Work done on the particle, when its displacement 0 to 5 m is



(1) 35 J (2) 25 J

(3) 15 J (4) 5 J

5. A man of mass 50 kg is standing in an elevator. If elevator is moving up with an acceleration $\frac{g}{3}$ then work done by normal reaction of floor of elevator on man when elevator moves by a distance 12 m is (g = 10 m/s^2)

(1) 2000 J (2) 4000 J

(3) 6000 J (4) 8000 J

6. A particle moves along x-axis under the action of a position dependent force $F = (5x^2 - 2x)$ N. Work done by forces on the particle when it moves from origin to x = 3 m is

(1) 45 J (2) 36 J

(3) 32 J (4) 42 J

7. The angles between the force and the displacement for maximum and minimum work respectively be

(1) 90° and 0° (2) 0° and 90°

(3) 0° and 180° (4) 180° and 0°

8. A body constrained to move along the z-axis, under a constant force $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ N. Work done by this force in moving the body a distance 4 m along the z-axis is

(1) 12 J (2) 4 J

(3) 2 J (4) 6 J

Try Yourself

17. A body of mass 2 kg moving on a horizontal surface with speed $v_i = 1\ \text{ms}^{-1}$ enters a rough patch ranging from $x = 0.5$ m to $x = 1.5$ m. If the body comes to rest after crossing the patch, then find the retarding force on the body.

    Hint : W-E theorem,

    $$f_r \cdot s = \frac{1}{2}m(v_f^2 - v_i^2)$$

18. The velocity (v) of a particle of mass m moving along x-axis is given by $v = b\sqrt{x}$, where b is a constant. Find work done by the force acting on the particle during its motion from $x = 0$ to $x = 4$ m.

    Hint : $v_i = b\sqrt{0} = 0$

    $$v_f = b\sqrt{4} = 2b$$

    $$W = K_f - K_i$$

    $$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Knowledge Cloud

- The work done by a varying force during a round trip is not necessarily zero.
- The kinetic energy can never be negative because mass as well as the square of speed can never be negative. Its units are same as that of work. It is a scalar quantity. It does not depend on the particles direction of motion.
- If net force on a system is zero, then the net work done by different forces acting on system may be non-zero *e.g.*, when a bullet is fired from a gun, the net force on system of bullet and gun is zero but the kinetic energy of the system increases, which is equal to work done.

THE CONCEPT OF POTENTIAL ENERGY

The word potential suggests capacity or possibility for action. This term potential energy brings to one's mind 'stored energy'. A wound up spring or a stretched bow string possesses potential energy.

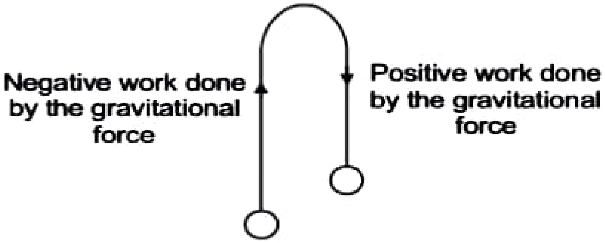

Negative work done by the gravitational force
Positive work done by the gravitational force

Fig. : 15

Let us consider throwing a ball upward. As it rises, the work done on the ball by the gravitational force, W_g is negative as the force transfers energy **from** the kinetic energy of the ball. This energy is transferred by the gravitational force **to** the potential energy of the ball-earth system. The ball shows, stops and then begins to fall back. Now, the work done, W_g, on the ball by the gravitational force is positive-that force transfers energy **from** the gravitational potential energy of the ball-earth system **to** the kinetic energy of the ball. In either case, the change ΔU in gravitational potential energy is defined as being equal to negative of the work done on the ball by the gravitational force (W).

$$\Delta U = -W \qquad \text{...(19)}$$

This also applies to a block spring system, as in Fig. 16

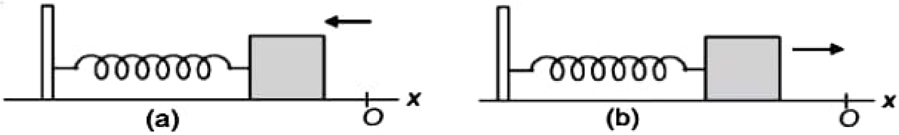


Fig. : 16

[Fig. 16(a) A block attached to a spring and initially at rest at $x = 0$ is set in motion towards the left. As the block moves leftward, spring force does negative work on it, Fig. 16(b) As the block moves towards $x = 0$, the spring force does positive work on it]

In Fig. 16(a), the spring force acts rightward and thus does negative work on the block, transferring energy from the kinetic energy of the block to the elastic potential energy of spring-block system. The block slows and stops finally and then begins to move rightward as the spring force is still rightward. The transfer of energy is now from potential energy of the spring-block system to kinetic energy of the block.

The gravitational force on a ball of mass m is mg, g is regarded as constant near the Earth's surface (*i.e.*, height 'h' of the ball above the Earth's surface is very small as compared to Earth's radius R ($h << R$)) so that variation of g is ignored. Lets take the upward direction as positive. Consider raising a ball of mass m upto a height 'h'. The work done by the external agency against the gravitational force is mgh, which gets stored as potential energy. Gravitational potential energy as a function of the height h, is denoted by U_h and it is the negative of work done by the gravitational force in raising the body to that height.

$$U_h = mgh$$

If h is taken as a variable, gravitational force F equals the negative of the derivative of U_h w.r.t. h.

$$F = -\frac{d}{dh}U_h = -mg$$

The negative sign indicates that the gravitational force is downward. On releasing, ball comes down with increasing speed. If v is the speed just before it hits the ground, then

$$v^2 = 2gh$$

Multiplying both sides of this equation with $\frac{1}{2}m$, we get,

$$\frac{1}{2}mv^2 = mgh$$

Which shows that the gravitational potential energy of the body at height h, manifests itself as kinetic energy on reaching the ground. Physically, this notion is applicable only to the class of forces where work done against the force gets 'stored up' as energy. It manifests itself as kinetic energy when external constraints are removed. Mathematically, the potential energy $V(x)$ (for simplicity, in one dimension) is defined if the force $F(x)$ can be written as

$$F(x) = -\frac{dV}{dx}$$

$$\int_{x_i}^{x_f} F(x)dx = -\int_{V_i}^{V_f} dV = V_i - V_f$$

The work done by a **conservative force** such as gravity or the spring force depends on the initial and final positions only, *e.g.*, an object of mass m is released from rest, from the top of a smooth (frictionless) inclined plane of height h, its speed at the bottom = $\sqrt{2gh}$ irrespective of the angle of inclination (it does not depend on path taken). If the work done or the kinetic energy depends on other factors such as the velocity or the path taken by the body, the force is known as **non-conservative force.**

Dimensions of energy : $[ML^2T^{-2}]$ (same for KE, PE and work)

SI unit of potential energy : Joule, the same as kinetic energy or work. Commercial unit of energy is kWh, 1 kWh = 3.6×10^6 J

To reiterate, the change in potential energy, for a conservative force, ΔV is equal to the negative of the work done by the force.

$$\Delta U = -F(x)\Delta x \qquad \ldots(20)$$

We saw in the example of falling ball, how potential energy was converted to kinetic energy. We shall now examine an important principle of conservation in mechanics.

THE CONSERVATION OF MECHANICAL ENERGY

For simplicity, we are considering one-dimensional motion. Suppose that a body undergoes displacement Δx under the action of a conservative force F. From the W-E theorem,

$$\Delta K = F(x)\Delta x$$

If the force is conservative, the potential energy function U_x can be defined such that

$$-\Delta U = F(x)\Delta x \qquad \text{from eq. (20)}$$

Adding the above two equations, we get

$$\Delta K + \Delta U = 0$$

or $\Delta(K + U) = 0$ ...(21)

$\Rightarrow$ **$K + U$, the sum of the kinetic and potential energies of the body is a constant.**

Over the whole path x_i to x_f

$$K_i + U_{x_i} = K_f + U_{x_f} \qquad \ldots(22)$$

The quantity $K + Ux$ is the total mechanical energy of the system. Kinetic energy K and the potential energy Ux may charge individually from point to point, but the sum is a constant.

Some definitions of a conservative force

(1) A force $F(x)$ is conservative if it can be derived from a scalar quantity U_x by the relation $\Delta U = -F(x)\Delta x$.

(2) The work done by the conservative force depends only on the end points.

$W = K_f - K_i = (U_{x_i} - U_{x_f})$ this depends on the end points.

(3) Work done by a conservative force in a closed path is zero. This is apparent from equation (22) as $x_i = x_f$

Thus, the principle of conservation of total mechanical energy can be stated as

"The total mechanical energy of a system is conserved if the forces doing work on it, are conservative".

Let us consider a ball of mass m being dropped from a cliff of height H.

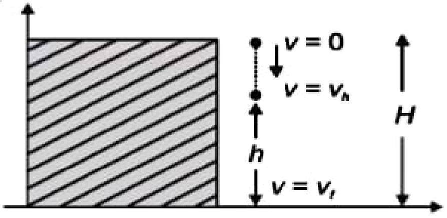


Fig. 17 : The conversion of PE to KE for a ball of mass m dropped from height H.

Total mechanical energies E_0, E_h and E_H at the indicated heights zero (ground level), h and H, are

$$E_H = mgH \quad \text{...(23)} \quad \text{(PE at height } H \text{ as KE = 0)}$$

$$E_h = mgh + \frac{1}{2}mv_h^2 \quad \text{...(24)} \quad \left(\text{Sum of PE and KE, } PE_h = mgh, KE_h = \frac{1}{2}mv_h^2\right)$$

$$E_0 = \frac{1}{2}mv_f^2 \quad \text{...(25)} \quad \text{(KE at the ground as PE = 0)}$$

Where v_h and v_f are velocities at height h and at the ground.

The constant force is a special case of a spatially dependent force $F(x)$. Hence, the mechanical energy is conserved thus, $E_H = E_0$

or $$mgH = \frac{1}{2}mv_f^2$$

$$v_f = \sqrt{2gH}$$

A result that we got earlier for a freely falling body.

Further, $E_H = E_h$

$$\Rightarrow \quad mgH = mgh + \frac{1}{2}mv_h^2$$

or $$v_h^2 = 2g(H-h) \quad \text{...(26)}$$

A familiar result from kinematics.

At height H, the energy is purely potential (KE = 0).

At height h, the energy is partially converted to kinetic.

At ground level the energy is fully kinetic (PE = 0).

This illustrates the conservation of mechanical energy.

Conservative and Non-Conservative Forces

From equation (A)

$$U_f - U_i = -\int \vec{F} \cdot d\vec{r}$$

That is, the change in P.E. = negative of the work done by the internal force $\vec{F}$ on the system.

"This special type of internal force of a system" (the work done *against* which is stored in the system as its potential energy which can be recovered later), is called conservative force.

All internal forces of a system are not conservative. For example you may reconsider the situations discussed in figure when the horizontal surface on which block moves, is rough. In this case, when the block moves from *A* to *B*; and again comes to *A*, the final kinetic energy of the system [figure (c)] is less than that of the system in initial configuration [figure (a)]. This means energy is lost in doing work against the friction force between the horizontal floor and the block. If we wait, the energy continuously reduces to zero.

Here, it should be noted that in the absence of friction, the system regains its kinetic energy as it returns to its initial configuration. This implies that the **work done by the spring force (a conservative force) in a round trip of a system is zero**. But in the presence of friction, the **work done by friction on the system in a round trip is negative (not zero)**.

"This (friction) type of *internal force of a system* (the work done *against* which is not stored in the system as its potential energy which can be recovered later), is called non-conservative force".

Properties of Conservative and Non-conservative Forces

Conservative forces	Non-conservative forces
1. It can always be expressed as negative of the gradient of the potential energy *i.e.,* $F = \frac{-dU}{dr}$.	1. It has no relation with the potential energy.
2. The work done by it is reversible.	2. The work done by it is not reversible.
3. The work done by it is independent of the path of the body and depends only on the starting and ending point.	3. The work done by it is not independent of the path of the body.
4. When the starting and end points are same, the total work is zero *i.e.,* the work done in a round trip is zero.	4. When the starting and end points are same, the total work is not necessarily zero.

Example 9 : *Show that work done by a conservative force on a particle moving between two points is path independent.*

Solution :

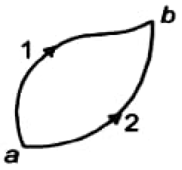


Fig. 18(a): A particle can move from *a* to *b* along either path 1 or path 2

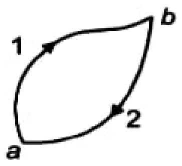


Fig. 18(b): The particle moves in a round trip from *a* to *b* via path 1 and then back from *b* to *a* along path 2

The force does work on the particle as it moves along each path.

$W_{ab,\,1}$ = work done from *a* to *b* along path 1

$W_{ab,\,2}$ = work done from *b* to *a* along path 2

If the force is conservative, then the net work done during the round trip must be zero.

$$W_{ab,\,1} + W_{ba,\,2} = 0$$

$$W_{ab,\,1} = -\,W_{ba,\,2} \qquad \text{...(i)}$$

Work done along the forward path must be the negative of the work done along the path back.

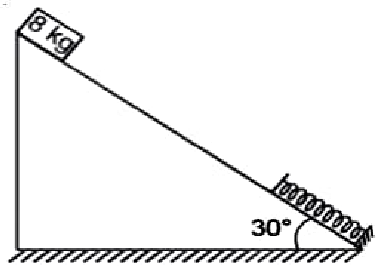
Let us now consider work $W_{ab,\,2}$ work done on the particle by the force when the particle moves from *a* to *b* along path 2, as shown in figure 18(a). If the force is conservative, then work is the negative of $W_{ba,\,2}$

$$W_{ab,\,2} = -\,W_{ba,\,2} \qquad \text{...(ii)}$$

From (i), (ii), we get

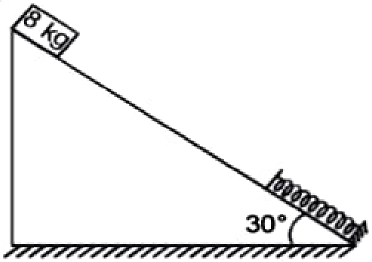
$$\boxed{W_{ab,1} = W_{ab,2}}$$

Example 10 : *A block of mass 8 kg is released from the top of an inclined smooth surface as shown in figure. If spring constant of spring is 200 N/m and block comes to rest after compressing spring by 1 m then find the distance travelled by block before it comes to rest.*



Solution : Consider d is the distance travelled by block along the plane and height it come in downward direction $h = d\sin 30°$

Now, when block will come to rest, The decrease in its potential energy will store in spring



$$\therefore \quad mgh = \frac{1}{2}ky^2 \qquad [y \text{ is compression in spring}]$$

$$\Rightarrow \quad mg\, d\sin 30° = \frac{1}{2}ky^2$$

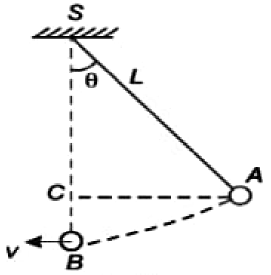
$$\Rightarrow \quad 8\times 10\times\frac{d}{2} = \frac{1}{2}\times 200\times 1^2$$

$$\Rightarrow \quad 40\,d = 100$$

$$\Rightarrow \quad d = 2.5 \text{ m}$$

Example 11 : *A simple pendulum of mass m and length l is held in position at an angle θ with vertical. If it is released from this position, find the speed with which it will pass the lowest position.*

Solution : During motion of pendulum from A to B, its potential energy will convert into kinetic energy.



The height pendulum comes in downward direction is BC

and $BC = SB - SC = L - L\cos\theta$

By law of conservation of mechanical energy,

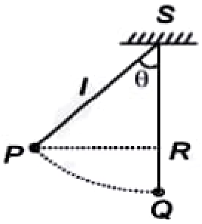
$$mgh = \frac{1}{2}mv^2$$

$$\Rightarrow \quad mg[L - L\cos\theta] = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{2gL[1-\cos\theta]}$$

Try Yourself

19. A simple pendulum with bob of mass m and length l is held in position at an angle θ with the vertical. Find its speed when it passes the lowest position.



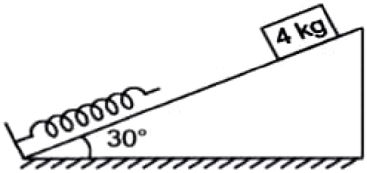
Hint : The height bob comes down is QR.

$h = QR = SQ - SR = l - l\cos\theta$

By law of conservation of mechanical energy.

$$mgh = \frac{1}{2}mv^2$$

20. A block of mass 4 kg is released from the top of an inclined smooth surface as shown in figure. If spring constant of spring is 100 N/m and block comes to rest after compressing spring by 1 m, find the distance travelled by the block before it comes to rest.



Hint : Let d be the distance travelled by block along the plane. Height it comes down in downward direction, $h = d\sin30°$.

When block comes to rest, decrease in its PE will be stored in the spring.

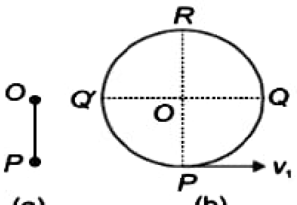
$\therefore \ mgh = \frac{1}{2}ky^2$ (y is the compression in the spring)

21. The potential energy of a particle of mass 1 kg moving along x-axis given by $U(x) = \left[\frac{x^2}{2} - x\right]$ J.

If total mechanical energy of particle is 2 J, then find its maximum speed.

22. If a simple pendulum of length l and mass m has maximum angular displacement θ, then find its maximum kinetic energy.

Motion in a Vertical Circle



(a) (b)

Fig. : 19

Let a particle P be suspended in a vertical plane, by a massless, inextensible string from a fixed point O. In Figure 19(a), the string is vertical with P vertically below the point of suspension O. The particle is in equilibrium. Let the particle be given an initial velocity $\vec{v}_1$ in horizontal direction, as shown in figure 19(b). The particle moves along a vertical circle (radius = length of the string). The point of suspension is the centre of the circle.

This motion has to be **non-uniform circular motion**. **Velocity** of the particle changes both in magnitude and direction. Speed of particle **decreases** continuously as it moves up the circle (*i.e.*, from $P \to Q \to R$) due to the work done **against** the force of gravity speed of particle **increases** continuously as it moves down the circle (*i.e.*, from $R \to Q' \to P$) due to work done by the force of gravity on the particle.

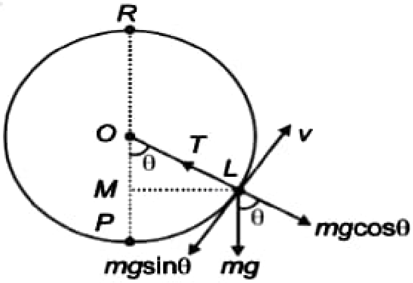


Fig. : 20

To get the basic characteristics of vertical circular motion, consider an instantaneous position of a particle, say at *L*. This is shown in Fig. 20, where the string makes an angles θ with the vertical line *OP*. At this position, the forces acting on the particle are

(i) Weight (= *mg*) acting vertically downwards

(ii) Tension (= *T*) acting along *LO* (in the string)

Direction of the instantaneous velocity *v* is along the tangent to the circle at *L*. The corresponding instantaneous centripetal force = $\frac{mv^2}{r}$ [where *r* (= length of string *l*) is radius of the circular path.] acts along *LO*.

Taking components of mg, mg cosθ acts opposite to *LO* and *mg* sinθ acts opposite to $\vec{v}$ (along tangent to the circular path at *L*, *i.e.*, perpendicular to *LO*).

The net force towards centre of the circle (along *LO*) = $T - mg\cos\theta$ this is necessary centripetal force $\left(=\frac{mv^2}{r}\right)$.

So, $\frac{mv^2}{r} = T - mg\cos\theta$

$$\therefore \quad T = \frac{mv^2}{r} + mg\cos\theta \qquad \ldots(27)$$

Taking horizontal direction at the lowest point *P*, as the position of zero gravitational potential energy, as per the law of conservation of energy

Total energy at *P* = Total energy at *L*

$$\therefore \quad \frac{1}{2}mv_1^2 + 0 = \frac{1}{2}mv^2 + mgh \qquad \ldots(28)$$

Where *MP* = *h*, is the vertical height of the particle above *P*. *OM* = *OL*cosθ = *r*cosθ (from right angled Δ*OML*)

$\therefore \quad MP = h = OP - OM = r - r\cos\theta = r(1 - \cos\theta)$

or $\quad h = r(1 - \cos\theta) \qquad \ldots(29)$

From equations (28) & (29),

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv^2 + mgr(1-\cos\theta)$$

or $\quad v_1^2 = v^2 + 2gr(1-\cos\theta) \qquad \ldots(30)$

Put the value of v^2 from equation (30) in equation (27)

$$T = \frac{m}{r}[v_1^2 - 2gr(1-\cos\theta)] + mg\cos\theta$$

$$T = \frac{mv_1^2}{r} - 2mg(1-\cos\theta) + mg\cos\theta$$

$$T = \frac{mv_1^2}{r} - 2mg + 3mg\cos\theta \qquad \text{...(31)}$$

This equation gives tension T as a function of θ. We can use it to see the details when the particle is at (i) lowest (P) (ii) mid way (horizontal) (Q) and (iii) highest position (R)

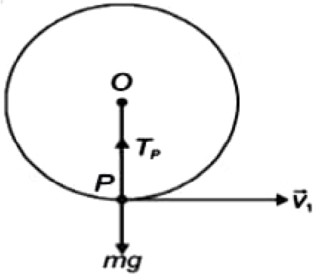


Fig. : 21

At P, $\theta = 0$

The tension in the string = T_P

$$T_P = \frac{mv_1^2}{r} - 2mg + 3mg\cos 0° \text{ (from equation 31)}$$

$$T_P = \frac{mv_1^2}{r} + mg \qquad \text{...(32)}$$

At Q, $\theta = 90°$ string is in horizontal position (momentarily).

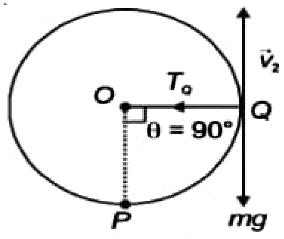


Fig. : 22

Let $\vec{v}_2$ be the instantaneous velocity at Q and let T_Q be the instantaneous tension is the string.

Using equation (31)

$$T_Q = \frac{mv_1^2}{r} - 2mg + 3\,mg\cos 90°$$

$$T_Q = \frac{mv_1^2}{r} - 2mg \qquad \text{...(33)}$$

The change in the tension, as the particle moves from P to Q

$$= T_P - T_Q = \left(\frac{mv_1^2}{r} + mg\right) - \left(\frac{mv_1^2}{r} - 2mg\right) \qquad \text{Using equations (32) and (33)}$$

$T_P - T_Q = 3\ mg$

At highest point R, $\theta = 180°$

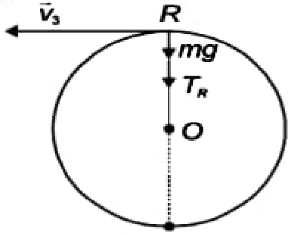


Fig. : 23

Let $\vec{v}_3$ be the instantaneous velocity at R and let T_R the instantaneous tension in the string. Using equation (31).

$$T_R = \frac{mv_1^2}{r} - 2mg + 3mg\cos 180°$$

$$T_R = \frac{mv_1^2}{r} - 5mg \qquad \text{...(34)}$$

The change in tension in the string as the particle moves from P to R

$$= T_P - T_R$$

$$= \left(\frac{mv_1^2}{r} + mg\right) - \left(\frac{mv_1^2}{r} - 5\,mg\right) \qquad \text{Using equations (32) \& (34)}$$

$T_P - T_R = 6$ mg

⇒ The tension in the string is **maximum** at **lowest point** P.

And The tension in the string is **minimum** at **highest point** R.

This is so because at the highest point, a part of centripetal force, needed to keep the particle moving in circular path is provided by weight (mg) of the particle.

From equation (34), we find that T_R can be (i) positive (ii) negative or (iii) zero, depending on the value of v_1. When T_R becomes negative, **string slackens** and the particle will fall down before completing its circular path.

⇒ Minimum value of T_R should be zero for completing the vertical circle

So, $$(T_R)_{\min} = \frac{m(v_1)^2_{\min}}{r} - 5\text{ mg} = 0 \qquad \text{from equation (34)}$$

$$\therefore \quad (v_1)_{\min} = \sqrt{5gr} \qquad \text{...(35)}$$

Using equation (30), the minimum speed, which the particle must have at the highest point R, so that it completes the vertical circle, is given by

$$(v_1)^2_{\min} = (v_3)^2_{\min} + 2gr(1 - \cos 180^\circ)(\text{as } \theta = 180^\circ \text{ at } R)$$

$$5gr = (v_3)^2_{\min} + 4gr \qquad \text{Using equation (35)}$$

or $$(v_3)_{\min} = \sqrt{gr} \qquad \text{...(36)}$$

When the particle completes its motion along the vertical circle, it is called '**looping the loop**'. For this, **minimum speed at the lowest point must be** $\sqrt{5gr}$.

Let us calculate tension in the string, when the particle is just able to do 'looping the loop' corresponding to

$$v_1 = (v_1)_{\min} = \sqrt{5gr}$$

From equation (32)

$$T_P = \frac{m}{r}(5gr) + mg = 6\,mg \qquad \text{...(37)}$$

From equation (34)

$$T_R = \frac{m}{r}(5gr) - 5mg = 0 \qquad \text{...(38)}$$

Note :

(1) *If* $v_0 \geq \sqrt{5gl}$, *the bob will complete full circular path.*

(2) *If* $v_0 = \sqrt{5gl}$, *the tension at the top is zero but the velocity is* $\sqrt{gl} > 0$.

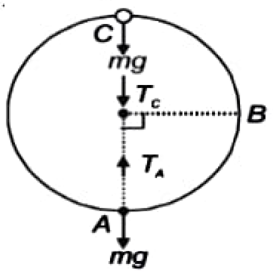
(3) *If* $\sqrt{2gl} < v_0 < \sqrt{5gl}$, *the bob leaves the circular trajectory when the tension in the string is zero but the speed of the bob is not zero. When the bob leaves the circular trajectory,* $90^\circ < \theta < 180^\circ$, *where* θ *is the angular displacement from the lowest position of the bob.*

(4) *If* $0 < v_0 \leq \sqrt{2gl}$, *the bob will oscillate about the lowest position having maximum angular displacement* θ_0 ($\leq 90^\circ$). *For* $\theta_0 < 90^\circ$, *the speed will be zero at the extreme position but the tension will not be zero. For* $\theta_0 = 90^\circ$, *the speed as well as tension will be zero.*

Your Chapter in Daily Life

1. A circus acrobat, performing in the '**circle of death**', speeds up his motor cycle inside the circular cage before going into a **vertical loop**. When he acquires a speed > $\sqrt{5gr}$, at the lowest point, he would not fall down even when he is 'upside down' at the highest point, This is because, the centrifugal force on him > his weight at the highest point.
2. The pilot of an aircraft can successfully loop a vertical circle of radius *r* if his velocity at the lowest point > $\sqrt{5gr}$.

Example 12 : *A bob of mass m suspended by a massless inextensible string is rotated in a vertical circle of radius r as shown in the figure. Obtain an expression for speeds and kinetic energies at A, B and C. Comment on the nature of trajectory of the bob after it reaches the point C.*



Solution : Let horizontal velocity at lowest point *A* be V_A such that string becomes slack only on reaching highest point *C*. Taking the potential energy of the system to be zero at *A*, the total mechanical energy, *E*

At *A* :

$$E = \frac{1}{2}mv_A^2 \quad \text{...(i)} \quad (E = KE + PE = KE \text{ as } PE = 0)$$

$$T_A - mg = \frac{mv_A^2}{r} \quad \text{...(ii)} \quad (\text{Using Newton's II}^{nd} \text{ Law})$$

Where T_A is the tension in the string at *A*.

At the highest point *C*, the string slackens, so $T_C = 0$

At *C* :

$$E = \frac{1}{2}mv_C^2 + 2mgr \quad \text{...(iii)} \quad (\text{Where } v_C \text{ is the velocity at } C.)$$

$$mg = \frac{mv_C^2}{r} \quad \text{...(iv)} \quad (\text{Newton's II}^{nd} \text{ law})$$

From equation (iv), $\frac{1}{2}mv_C^2 = \frac{1}{2}mgr$

Put this value in eq. (iii)

$$E = \frac{1}{2}mgr + 2mgr$$

$$E = \frac{5}{2}mgr \quad \text{...(v)} \quad (\text{This is also KE at } A \text{ as } PE_A = 0)$$

From equations (i) & (v)

$$\frac{1}{2}mv_A^2 = \frac{5}{2}mgr$$

$$v_A = \sqrt{5gr} \quad \text{...(vi)}$$

$$\boxed{\text{Velocity at } A = \sqrt{5gr}}$$

$$\boxed{\text{Kinetic energy at } A = \frac{5}{2}mgr} \quad \text{...(vii)}$$

From equation (iv), $\boxed{v_c = \sqrt{gr}}$ ...(viii)

$$\boxed{\text{Kinetic energy at } C = \frac{1}{2}mgr} \quad \text{...(ix)}$$

At *B* :

$$E = \frac{1}{2}mv_B^2 + mgr \quad \text{...(x)}$$

(Where v_B is the velocity at *B*)

This should be same as the energy at *A*.

$$\text{So, } \frac{1}{2}mv_B^2 + mgr = \frac{1}{2}mv_A^2 = \frac{5}{2}mgr \quad \text{(Using equation (vii))}$$

$$\frac{1}{2}mv_B^2 = \frac{3}{2}mgr$$

$$\Rightarrow \quad \boxed{v_B = \sqrt{3gr}} \quad \text{...(xi)}$$

$$\boxed{\text{Kinetic energy at } B = \frac{3}{2}mgr} \quad \text{...(xii)}$$

At *C*, the string becomes slack and velocity of bob is horizontal and to the left, Bob continues on its circular path and completes the revolution.

Try Yourself

23. A small stone of mass 100 g in rotated in a vertical circle of radius 40 cm. What is the minimum speed needed at the lowest point for looping the loop? Also find the tension in the string at this point. (g = 10 ms^{-2}).

Hint : $v_{\text{min}} = \sqrt{5gr}$

$$T_1 = \frac{mv_{\text{min}}^2}{r} + mg = 6\,mg$$

24. A massless, inextensible string of length 1 m has a breaking strength of 1 kg wt. A stone of mass 0.2 kg, tied to one end of the string is made to move in a vertical circle. Can the stone describe the vertical circle? (g = 10 ms^{-2}).

Hint : T_{max} = 1 kg wt

= 1 × 10 = 10 N

Let tension at lowest point be *T*.

T_{min} = 6 mg

Example 13 : *A small stone of mass 0.4 kg tied to a massless inextensible string is made to loop the loop. Radius of the path is 4 m. Find its speed at the highest point. How would this speed change if mass of the stone is decreased by 10%? ($g = 10\ ms^{-2}$).*

Solution : Speed at highest point = $\sqrt{gr} = \sqrt{10\times4} = \boxed{6.32\ ms^{-1}}$. It **does not depend** on the **mass** of the stone. So, this speed **remains the same** on changing mass of the stone.

Try Yourself

25. A particle of mass 100 g, is made to describe a vertical circle of radius 1 m. Its instantaneous speed is 1 ms^{-1} when the string makes an angle of 30° with the vertical. Find the tension in the string at this position. Can the particle complete its circular path? ($g = 10\ ms^{-2}$)]

Hint : Tension in the string when it makes an angle θ with the vertical is

$$T = \frac{mv^2}{r} + mg\cos\theta$$

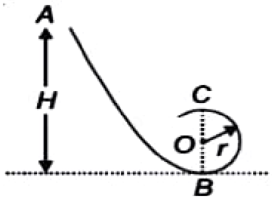
Let speed at the lowest point be v_1

$$v_1^2 = v^2 + 2gr(1-\cos\theta)$$

$(v_1)_{min} = \sqrt{5gr}$ check if $v_1 < (v_1)_{min}$.

If yes, then it would not complete its vertical circular path.

26. A particle of mass m is released from point A as shown in the figure. Would it 'loop the loop' if $H = 4r$? What is the force on the circular track when it is at point (a) B, (b) C?



Hint : Let speed at the lowest point B be v_B.

Total energy at A = Total energy at B

$$0 + mgH = \frac{1}{2}mv_B^2$$

$$v_B = \sqrt{2gH} = \sqrt{8gr}$$

Let N_1 be the force exerted by the track at B.

$$N_1 = mg + \frac{mv_B^2}{r}$$

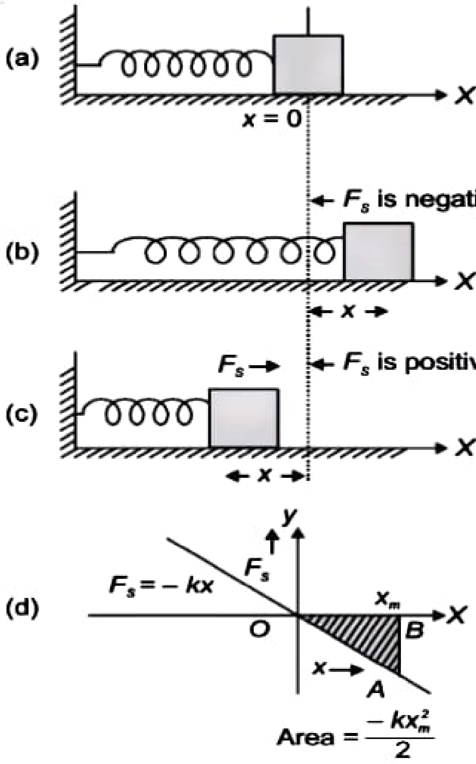
If v_C is velocity at C.

$$v_C^2 = v_B^2 - 4gr$$

N_2 is force exerted by the track at C.

$$N_2 = \frac{mv_C^2}{r} - mg$$

THE POTENTIAL ENERGY OF A SPRING


(a)
X
x = 0
(b)
F_s is negative, x is positive
X
x
(c)
F_s
F_s is positive, x is negative
X
x
(d)
y
F_s
$F_s = -kx$
x_m
X
O
B
x
A
Area = $\frac{-kx_m^2}{2}$

The spring force $F_s = 0$, when x = displacement for equilibrium position = 0

For the stretched spring $x > 0$ and $F_s < 0$

For the compressed spring $x < 0$ and $F_s > 0$

For the plot of F_s vs x, the area of the shaded triangle represents the work done by the spring force. $W_s = -\frac{kx_m^2}{2}$ (–ve as F_s and x have opposite sign)

Fig. : 24

Fig. 24(a) shows a block attached to a spring and resting on a smooth horizontal surface. Let us make two assumptions about the spring (i) It is massless *i.e.*, its mass is negligible as compared to mass of the block (ii) It is an **ideal spring** *i.e.*, it obeys Hooke's law exactly *i.e.*, the spring force $F_s \propto x$ where x is the displacement of the block from its equilibrium position. x can be either positive [Fig. 24(b)] or negative [Fig. 24(c)]. The force law for the spring is called Hooke's law *i.e.*, $F_s = -kx$. The constant k is called the spring constant. Its SI unit is Nm^{-1}. If k is large, the spring is said to be stiff and if k is small, the spring is said to be soft.

Let us also assume that contact between the block and the floor is frictionless and the block is particle-like. Let us give the block a rightward jerk to get it moving. The spring force F_s does work on the block decreasing the KE and slowing it. We **cannot** find this work by using $W = Fd\cos\theta$ as this equation assumes a constant force. The spring force is a variable force. Let the block's initial position be x_i and its later position x_f. Divide the distance between these positions into many segments each of small length Δx. Label them, starting from x_i, as 1, 2, ... and so on. As the block moves through a segment the spring force hardly changes as the segment is so short that x hardly changes. So, we can approximate the force magnitude as constant within the segment. Label their magnitudes as F_{x_1} in segment-1, F_{x_2} in segment-2, and so on. Suppose we pull the block outward as in Fig.24(b) and the extension is x_m. Work done by the spring force is

$$W_s = \int_0^{x_m} F_s dx \qquad \text{(Replacing the summation by integral as } \Delta x \to 0)$$

$$= -\int_0^{x_m} kx dx \qquad \text{(Where } F_s = -kx)$$

$$W_s = \frac{-kx_m^2}{2} \qquad \ldots(39)$$

You can also get this expression by considering the area of the triangle as in Fig. 24(d). Work done by the external pulling force F is positive as it overcomes the spring force. The same is true when the spring is compressed with the displacement x_c (< 0). The spring force does work $W_s = -\frac{kx_c^2}{2}$ while the external force F does work $+\frac{kx_c^2}{2}$. Consider moving the block from an initial displacement x_i to a final displacement x_f, the work done by the spring force W_s is

$$W_s = -\int_{x_i}^{x_f} kx\,dx = -\left[\frac{kx^2}{2}\right]_{x_i}^{x_f} = \frac{kx_i^2}{2} - \frac{kx_f^2}{2} \qquad \text{...(40)}$$

So, the work done by the spring force depends only on the end points. If the block is pulled from x_i and allowed to return to x_i :

$$W_s = -\int_{x_i}^{x_i} kx\,dx = \frac{kx_i^2}{2} - \frac{kx_i^2}{2} = 0 \qquad \text{...(41)}$$

The work done by the spring force in a cyclic process is zero.

The spring force (i) is position dependent only as stated by Hooke, ($F_s = -kx$), (ii) does work which only depends on the initial and final positions *e.g.*, equation (40)

⇒ **The spring force is a conservative force.**

The potential energy $V(x)$ of the spring is defined to be zero when block and spring system is in the equilibrium position. For an extension (or compression) x, the above analysis suggests that

$$V(x) = \frac{kx^2}{2}$$

$$\Rightarrow \quad -\frac{dV}{dx} = -\frac{d}{dx}\frac{kx^2}{2} = -kx, \text{ The spring force.}$$

If a block of mass m is extended to x_m and released from rest, its total mechanical energy at any point x. (where $-x_m < x < +x_m$) is given by

$$\frac{1}{2}kx_m^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

⇒ The speed and the KE will be maximum at the equilibrium position *i.e.*,

$$\frac{1}{2}mv_m^2 = \frac{1}{2}kx_m^2$$

Where v_m is the maximum speed

or $\quad v_m = \sqrt{\frac{k}{m}}x_m$

$\frac{k}{m}$ has the dimensions of [T^{-2}]. The KE gets converted to PE and vice versa. However, the total mechanical energy remains constant, as shown in Fig. 25

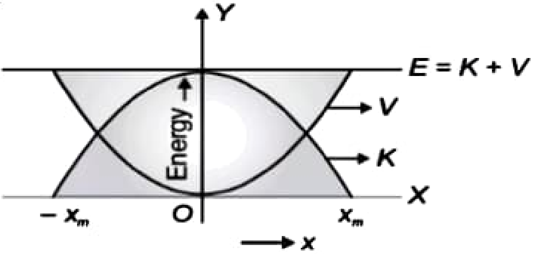


Fig. : 25

Parabolic plot of PE, V and KE, K of a block attached to a spring. The two plots are complementary, one decreasing as the other increases. Total mechanical energy $E = K + V$ remains constant.

At equilibrium position, $V = 0$ and K = maximum

At extreme positions, V = maximum and $K = 0$

Joining the points given by $(V + K)$ at these positions gives a straight line indicated by $E = K + V$ which is parallel to x-axis indicating that it is constant. As the block moves from equilibrium position to extreme position, V increases and K decreases, but $E = K + V$ remains constant

Example 14 : *Auto manufacturers study the collisions of moving cars with mounted springs of different spring constants. Consider a car of mass 1500 kg moving with a speed of 36 km/h on a smooth road and colliding with a horizontally mounted spring of spring constant 7.5×10^3 N m^{-1}. Find the maximum compression of the spring.*

Solution : At maximum compression, KE of car gets converted completely into PE of the spring.

$$\text{KE of car} = K = \frac{1}{2}mv^2 = \frac{1}{2}\times 1500\times 10\times 10 \qquad \left(v = 36\times\frac{5}{18} = 10\text{ m/s}\right)$$

$$= 7.5 \times 10^4 \text{ J}$$

At maximum compression x_m, PE, V of the spring = KE, K of car

$$V = \frac{1}{2}kx_m^2 = 7.5\times 10^4 \text{ J}$$

$$x_m = \sqrt{\frac{2\times 7.5\times 10^4}{7.5\times 10^3}} = \boxed{4.47\text{ m}}$$

Try Yourself

27. A block of mass 0.1 kg attached to a spring of spring constant 400 N/m is pulled rightward from $x_0 = 0$ to $x_1 = 15$ mm. Find the work done by spring force.

    Hint : $W_s = -\frac{1}{2}kx^2$

28. In the above question, find the work done by the spring force as the block is moved leftward from $x_1 = 15$ mm to $x_2 = -10$ mm.

    Hint : $x_i = +15$ mm

    $x_f = -10$ mm

    $W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$

We conclude this section by making a few remarks on conservative forces.

(1) Time was not considered in the above discussions. In the example, we did not calculate the time over which the compression occurred. For temporal information, a solution of Newton's II law is required for the system.

(2) Not all forces are conservative *e.g.*, friction is a non-conservative force. The modified principle of conservation of energy is discussed in Example 13.

(3) The zero of the PE is arbitrary (*i.e.*, it is set according to convenience) *e.g.*, unstretched spring had zero PE at $x = 0$. For constant gravitational force, $v = 0$ was taken on Earth's surface. Once it is set, it can not be changed.

Example 15 : *Consider example 14, taking the coefficient of friction, μ to be 0.4 and calculate the maximum compression of the spring. ($g = 10\ ms^{-2}$)*

Solution : Both the frictional force and the spring force act so as to oppose the compression of the spring as shown in figure.

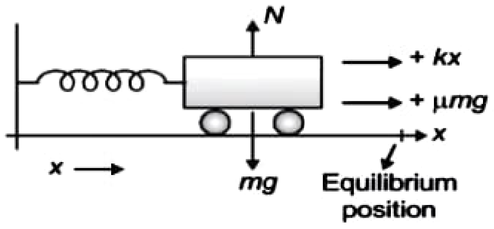


Fig. : The forces acting on the car

Use the work-energy theorem,

The change in KE = $\Delta K = K_f - K_i = 0 - \frac{1}{2}mv^2$

The work done by the net force is $W = -\frac{1}{2}Kx_m^2 - \mu mgx_m$

Equating the two, we get

$$\frac{1}{2}mv^2 = \frac{1}{2}Kx_m^2 + \mu mgx_m$$

$\mu mg = 0.4 \times 1500 \times 10 = 6000$ N

$$Kx_m^2 + 2\mu mgx_m - mv^2 = 0$$ (After rearranging the given equation)

$$x_m = \frac{-2\mu mg + \sqrt{4\mu^2m^2g^2 - 4(k)(-mv^2)}}{2K}$$ (Taking +ve sign with square root as x_m is +ve)

$$= \frac{-\mu mg + \sqrt{\mu^2m^2g^2 + mkv^2}}{K}$$

$$= \frac{-0.4\times1500\times10 + \sqrt{(0.4\times1500\times10)^2 + 1500\times7.5\times10^3\times10^2}}{7.5\times10^3}$$

= 3.74 m

Which, as expected, is less than the result in Example 14. If the two forces on the body consist of a conservative force F_c and a non-conservative force F_{nc}, the conservation of mechanical energy formula will have to be modified. By the W-E theorem,

$(F_c + F_{nc})\Delta x = \Delta K$

But $F_c\Delta x = -\Delta V$

Hence, $\Delta(K + V) = F_{nc}\Delta x$

$\Delta E = F_{nc}\Delta x$

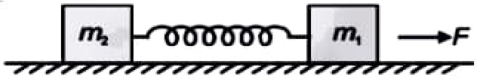
where E is the total mechanical energy. Over the path this assumes the form

$E_f - E_i = W_{nc}$

where W_{nc} is the total work done by the non-conservative forces over the path. Unlike conservative force, W_{nc} depends on the path taken.

Try Yourself

29. Two bodies of masses m_1 and m_2 are connected by a non-deformed light spring and lie on a horizontal plane. Coefficient of friction between the surface and the blocks is μ. Find an expression for the work done by the various forces. Force F is applied horizontally to body of mass m_1 in order to shift body of mass m_2.



Hint : Let the motion of m_2 start when the spring is stretched by x slowly. Work done by various forces will involve 3 terms. $F.x$, $(-\mu m_1 g)x$ and $-\frac{1}{2}kx^2$

30. For Q. 29, find the minimum constant force to be applied to body of mass m_1, in order to shift body of mass m_2. Take $m_1 = m_2 = 10$ kg, $\mu = 0.5$

Hint : $F.x = \mu m_1 gx - \frac{1}{2}kx^2 = 0$ ...(i)

But $Kx = \mu m_2 g$. (for just moving body of mass m_2)

Put the value of K in (i) to get F.

VARIOUS FORMS OF ENERGY: THE LAW OF CONSERVATION OF ENERGY

Energy comes in many forms which transform into one another.

Heat

A block of mass m sliding over a rough horizontal surface with speed v_0 comes to rest over a distance x_0. The work done by the force of kinetic friction f over x_0 is $-fx_0$.

By W-E theorem, $\frac{1}{2}mv_0^2 = fx_0$.

One point of view is that the kinetic energy of the block gets 'lost' due to frictional force. Actually, the work done by friction is transferred as heat energy.

Chemical Energy

It arises from the fact that the molecules participating in the chemical reaction have different binding energies. If total energy of the reactants is more than that of the products, heat is released and the reaction is known as **exothermic reaction**. If the reverse is true, heat is absorbed and the reaction is **endothermic**.

Electrical Energy

Energy is associated with an electric current. An urban Indian household consumes about 200 J of energy on an average.

The Equivalence of Mass and Energy

Einstein showed that mass and energy are equivalent and are related by the relation

$$E = mc^2 \quad ...(42)$$

Where c = speed of light in vacuum = 3×10^8 ms^{-1}

e.g., energy associated with 1 kg of matter

$E = 1 \times (3 \times 10^8)^2$ J $= 9 \times 10^{16}$ J.

This is equivalent to the annual electrical output of a large (3000 MW) power-generating station.

Nuclear Energy

In nuclear fission, a heavy nucleus like $^{235}_{92}U$, is split by a neutron into lighter nuclei. The final mass is less than the initial mass and the mass difference Δm gives the energy released according to the equation. $\Delta m = \frac{\Delta E}{c^2}$. In nuclear fusion (in the sun) four light hydrogen nuclei fuse to form a helium nucleus whose mass is less than the sum of masses of the reactants. This mass difference Δm is the source of energy $(\Delta m)c^2$.

Table-3 lists the total energies for a variety of events and phenomena.

Table-3 : Approximate energy associated with various phenomena

Description	Energy (J)
Big Bang	10^{68}
Radio energy emitted by the galaxy during its lifetime	10^{55}
Rotational energy of the milky way	10^{52}
Energy released in a supernova explosion	10^{44}
Ocean's hydrogen in fusion	10^{34}
Rotational energy of the earth	10^{29}
Annual solar energy incident on the earth	5×10^{24}
Annual wind energy dissipated near earth's surface	10^{22}
Annual global energy usage by human	3×10^{20}
Annual energy dissipated by the tides	10^{20}
Energy release of 15-megaton fusion bomb	10^{17}
Annual electrical output of large generating plant	10^{16}
Thunderstorm	10^{15}
Energy released in burning 1000 kg of coal	3×10^{10}
Kinetic energy of a large jet aircraft	10^{9}
Energy released in burning 1 litre of gasoline	3×10^{7}
Daily food intake of a human adult	10^{7}
Work done by a human heart per beat	0.5
Turning this page	10^{-3}
Flea hop	10^{-7}
Discharge of a single neuron	10^{-10}
Typical energy of a proton in a nucleus	10^{-13}
Typical energy of an electron in an atom	10^{-18}
Energy to break one bond in DNA	10^{-20}

Example 16 : *Examine table 3 and express (a) energy required to break one bond in DNA in eV, (b) typical energy of a proton in a nucleus in eV, (c) energy released in burning 1000 kg of coal in kilocalories.*

Solution : (a) Energy required to break one bond of DNA $= \frac{10^{-20}\,\text{J}}{1.6 \times 10^{-19}\,\text{J/eV}} \simeq 0.06\ \text{eV}$

(b) Typical energy of a proton in a nucleus $= \frac{10^{-13}\,\text{J}}{1.6 \times 10^{-19}\,\text{J/eV}} = 625000\ \text{eV}$

(c) Energy released in burning 1000 kg of coal $= \frac{3 \times 10^{10}\,\text{J}}{4.2 \times 10^{3}\,\text{J/kcal}} = 7142857.14\ \text{kcal}$

Try Yourself

31. Express the daily intake of a human adult in kilocalorie

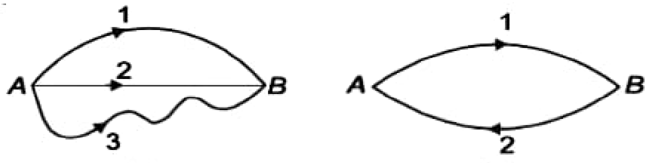
Hint : The average human consumption in a day = 10^7J
$1 \text{ cal} = 4.2 \text{ J}$
$1 \text{ kcal} = 4.2 \times 10^3 \text{ J}$

32. Express the energy of big bang in eV.

Hint : Energy of big bang = 10^{68} J
$1.6 \times 10^{-19} \text{ J} = 1 \text{ eV}$

The Principle of Conservation of Energy

Before discussing the principle of conservation of energy, let us discuss non-conservative forces in detail.



(a) (b)

Fig. : 26

A force is non-conservative if **work done** by, or against the force, is moving from one point A, in space, to another point B **depends on the path followed in moving from A to B.** In Fig. 26(a), let W_1, W_2 and W_3 denote the work done in moving a body from A to B along three different paths 1, 2 and 3 respectively. For a non-conservative force, $W_1 \neq W_2 \neq W_3$. Fig. 26(b) shows a particle moving along a **closed path** $A \rightarrow 1 \rightarrow B \rightarrow 2 \rightarrow A$. Let W_1 be the work done along the path $A \rightarrow 1 \rightarrow B$ and let W_2 be the work done along the path $B \rightarrow 2 \rightarrow A$. For a non-conservative force, $|W_1| \neq |W_2|$

$\therefore$ **In such a case, the net work done, along the closed path is not zero.**

Mathematically, $\oint \vec{F} \cdot d\vec{s} \neq 0$

i.e., for a non-conservative force, the work done along a closed path is not zero.

Two of the common examples are

(i) Frictional force

(ii) Viscous force

Non-conservative forces are usually **velocity dependent**. Consider a particle moving from A to B on a rough, horizontal surface. The force of friction is a non-conservative force.

Fig. : 27

Let W be the work done against friction in moving from A to B when it moves from B to A, W would again be the work done against force of friction. The net work done, against friction, in the round trip is 2 W. If this work is being done at the cost of kinetic energy, the loss of KE in the round trip = 2 W (numerically).

Consider next, a body having both KE (K) and PE (U) moving in a non-conservative force field. The total energy $E = (K + U)$ of the body, does not remain constant. Let E_i and E_f represents the total values of the initial and final energy. If W is the work done against non-conservative force, $E_i - E_f = W$.

The work done by non-conservative forces, appears in some other forms of energy, like heat, sound etc. when we take into account all forms of energy, the general **law of conservation of energy still holds good for conservative as well as non-conservative forces**. *e.g.*, a body of mass 0.1 kg has initial speed of 4 ms^{-1} at point A on a rough horizontal road. The coefficient of friction between the body and the road is 0.15. It moves to a point B at a distance of 2 m from A. (g = 10 ms^{-2}).

If the forces doing work on a system are conservative, the total mechanical energy of the system is conserved. If few forces are non-conservative, part of the mechanical energy may get converted into other forms such as heat, and light. But, total energy of an isolated system does not change (considering all forms of energy). Energy can neither be created, nor be destroyed.

Universe as a whole be regarded as an isolated system, so its total energy is constant. If its one part loses energy, other part must gain an equal amount of energy. The principle of conservation of energy cannot be proved. However, no violation of this principle has been observed.

Content Builder

Potential Energy Curve and Equilibrium System

If on a system or on an object a large number of forces act simultaneously in such a way that the resultant force on it is zero, then it is said to be in translatory equilibrium.

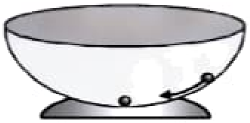
If the forces acting on the object are conservative and it is in equilibrium, then

$$F = 0 \quad \Rightarrow \quad -\frac{dU}{dr} = 0 \quad \text{or} \quad \frac{dU}{dr} = 0$$

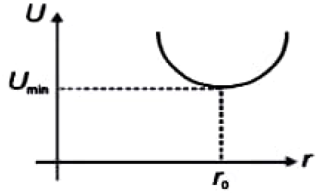
When field is conservative and object is in equilibrium, its potential energy is either minimum or maximum or constant.

On the basis of value of potential energy the equilibrium of object or system can be divided into three types.

1. **Stable equilibrium :** An object is said to be in stable equilibrium, if on slight displacement from equilibrium position it has tendency to come back to its original position. In case of stable equilibrium the potential energy is minimum. For example a small sphere placed in a smooth bowl as shown in figure. On displacing from the centre it will come back to its equilibrium position.

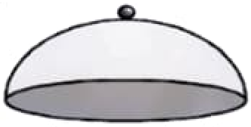


Stable equilibrium

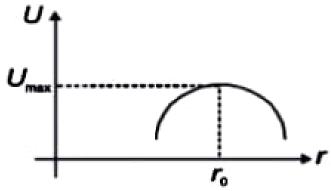


where r_0 is the stable equilibrium position.

2. **Unstable Equilibrium :** An object is said to be in unstable equilibrium, it on slight displacement from equilibrium position it moves in the direction of displacement. The potential energy of object is maximum in equilibrium state. For example a small sphere placed on a hemispherical shell as shown in figure.

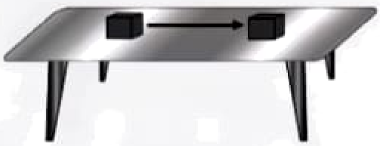


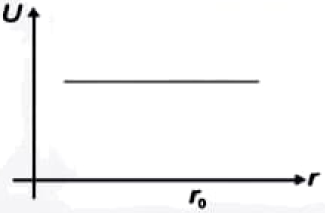
Unstable equllibrium



where r_0 is the unstable equilibrium position.

3. **Neutral equilibrium :** An object is said to be in neutral equilibrium state if on displacing from its equilibrium position it has no tendency to move in the direction of displacement or to come back to equilibrium position. For example a block placed on a horizontal table.





Example 17 : *A particle slides on frictionless x-y plane. It is acted on by a conservative force described by the potential-energy function* $U(x, y) = \frac{1}{2}k(x^2 + y^2)$. *Derive an expression for the force acting on the particle.*

Solution :

$$F_x = \frac{-\partial U}{\partial x} = -\frac{1}{2}k(2x + 0) = -kx$$

$$F_y = \frac{-\partial U}{\partial y} = -\frac{1}{2}k(0 + 2y) = -ky$$

$$\therefore \quad \vec{F} = F_x\hat{i} + F_y\hat{j} = -k(x\hat{i} + y\hat{j}) = -k\,\vec{r}$$

where $\vec{r}$ is the position vector of the particle. Hence $\vec{F} = -k\vec{r}$ is directed towards the origin.

Try Yourself

33. The potential energy $U(x)$ of a particle moving along x-axis is given by $U(x) = ax - bx^2$. Find the equilibrium position of particle.
34. The potential energy of an object of mass m moving in xy plane in a conservative field is given by $U = ax + by$, where x and y are position coordinates of the object. Find magnitude of its acceleration.

POWER

Power is defined as the time rate at which work is done or energy is transferred. The average power of a force is defined as the ratio of the work, W to the total time t taken

$$P_{av} = \frac{W}{t}$$

The instantaneous power is defined as the limiting value of the average power as time interval approaches zero.

$$P = \frac{dW}{dt} \qquad \ldots(43)$$

The work dW done by force $\vec{F}$ for a displacement $d\vec{r}$ is

$$dW = \vec{F} \cdot d\vec{r}$$

The instantaneous power can also be expressed as

$$P = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v} \qquad \ldots(43a)$$

where $\vec{v}$ is the instantaneous velocity when the force is $\vec{F}$.

Power is a scalar quantity like work and energy.

Its dimensions are $[ML^2T^{-3}]$.

Its SI unit is Watt (W), named after the Physicist James Watt, one of the innovators of the steam engine.

One watt of power is expanded when 1 joule of work is done per second *i.e.*, $1W = \dfrac{1\,J}{1\,s}$

Another unit of power is horse power (hp)

1 hp = 746 W

In British system, the unit of power is foot-pound per second.

$$\boxed{1\,\text{hp} = 550\ \text{ft lb/s} = 33{,}000\ \text{ft lb/min} = 746\ \text{W}}$$

$$\boxed{1\,\text{hp} = \frac{3}{4}\text{kW}}$$

We come across the unit watt while dealing with electrical goods like bulb, tube lights etc. *e.g.*, power of a bulb is said to be 60 W when it consumes 60 J of energy in one second.

Let us calculate the energy used by a 100 W bulb when it is on for 10 hours.

$$E = P \times t = \frac{100}{1000}\text{kW} \times 10\ \text{h} = 1\,\text{kWh} = 3.6 \times 10^6\ \text{J}$$

Our electricity bills show the energy consumption in units (kWh).

Note : kWh is commercial unit of energy. It is not a unit of power.

Example 18 : *An engine pumps 400 kg of water through height of 10 m in 40 s. Find the power of the engine if its efficiency is 80% ($g = 10\ ms^{-2}$).*

Solution : Work done by engine against gravity

$$W = mgh$$
$$= 400 \times 10 \times 10$$
$$= 40\ \text{kJ}$$

$$\text{Power used by engine} = \frac{W}{\Delta t}$$

$$= \frac{40 \times 10^3}{40}\text{W} = 1\,\text{kW}$$

If power of the engine is P, then

$$P \times \frac{80}{100} = 1\,\text{kW}$$

$$\Rightarrow \quad P = \frac{1000 \times 100}{80}\text{W}$$

$$\text{or} \quad \frac{100}{80}\text{kW} = \boxed{1.25\ \text{kW}}$$

Try Yourself

35. An engine pumps 800 kg of water through height of 10 m in 80 s. Find the power of the engine if its efficiency is 75%.

Hint : $W = mgh$ $\qquad P\times\frac{75}{100}=\frac{W}{\Delta t}$

36. The average work done by a human heart while it beats once is 0.5 J. Calculate the power used by heart if it beats 74 times in a minute.

$W = 0.5 \times 74$ $\qquad P=\frac{W}{t}$

Example 19 : *A particle is moving along x-axis under the action of a force, F which varies with its position (x) as $F \propto \frac{1}{\sqrt[4]{x}}$. Find the variation of power due to this force with x.*

Solution :

$$F \propto x^{-1/4}$$

$$\Rightarrow \quad a \propto x^{-1/4}$$

$$\Rightarrow \quad a = kx^{-1/4} \text{ (where } k \text{ is a proportionality constant)}$$

$$a=\frac{dv}{dt}=\frac{dx}{dt}\frac{dv}{dx}=kx^{-1/4}$$

$$\frac{vdv}{dx}=kx^{-1/4}$$

$$vdv = kx^{-1/4}dx$$

$$\int vdv = k\int x^{-1/4}dx$$

$$\frac{v^2}{2}=k\frac{x^{3/4}}{3/4}$$

$$v^2 \propto x^{3/4}$$

$$v \propto x^{3/8}$$

$$\therefore \quad P = Fv$$

$$P \propto \frac{x^{3/8}}{x^{1/4}}$$

$$\boxed{P \propto x^{1/8}}$$

Example 20 : *An elevator that can carry a maximum load of 1500 kg (elevator + passengers) is moving up with a constant speed of 2 ms^{-1}. The frictional force opposing the motion is 3000 N. Find the minimum power delivered by the motor to the elevator in watts as well as in horse power (g = 10 ms^{-2})*

Solution : The downward force on the elevator is

$$F = mg + F_f = (1500 \times 10) + 3000 = 18000 \text{ N}$$

The motor must supply enough power to balance this force.

Hence, $P=\vec{F}.\vec{v}=18000\times 2=\boxed{36000 \text{ W}}=\frac{36000}{746}=\boxed{48.3\ hp}$

Try Yourself

37. A particle is moving along *x*-axis under the action of a force *F*, which varies with its position (*x*) as $F \propto \frac{1}{\sqrt{x}}$. Find the variation of power due to this force with *x*.

Hint : $F \propto x^{-1/2}$ $\therefore \quad a = kx^{-1/2}$

$$a \text{ or } \frac{vdv}{dx} = kx^{-1/2}$$

$$\int vdv = k\int x^{-1/2}dx$$

Find *v*.

$$P = Fv$$

$$P \propto \frac{v}{\sqrt{x}}$$

38. The position (*x*) of a body moving along *x*-axis at time (*t*) is given by $x = 3t^2$, where *x* is in metre and *t* is in second. If mass of body is 2 kg, then find the instantaneous power delivered to body by force acting on it at *t* = 4 s.

Hint : Instantaneous power = $\vec{F}\cdot\vec{v}$

$$F = ma = \frac{mdv}{dt} = \frac{md}{dt}\left(\frac{dx}{dt}\right) \qquad x = 3t^2$$

Find $v\left(=\frac{dx}{dt}\right)$ then find $\vec{F}\cdot\vec{v}$.

39. A 10 metric ton truck drives up a hilly road of gradient 1 in 50 at a speed of 36 km h^{-1}. If the coefficient of kinetic friction between the road and tyres is 0.2, calculate the power delivered by the engine ($g = 10$ ms^{-2})

Hint : Weight, $W = mg$

Component of weight down the incline = $F = mg\sin\theta$

Frictional force, $F_r = \mu_k mg$

Total force, $F' = mg\sin\theta + F_r$

Power = $F'v$

40. For Q. 39, calculate how fast will the same truck move down the hill with same power.

Hint : Force on the truck when it moves down the hill is

$F'' = mg\sin\theta - \mu_k mg\cos\theta$

Power, $P = F''v'$ calculate v'.

41. Provided a racing car does not lose traction, the time taken by it to race from rest through a distance depends mainly on engine's power P. Derive the time t in terms of S and P, assuming that power is constant.

Hint : $P = mav = mv\frac{dv}{dt}$ $\left(a = \frac{dv}{dt}\right)$

$$\Rightarrow \int_0^v v dv = \frac{P}{m}\int_0^t dt$$

Find v, then $v = \frac{dx}{dt}$

$$\int_0^x dx = \int_0^t v dt$$

Find x.

EXERCISE

9. A cubical vessel of height 1 m is full of water. The minimum work done in taking water-out from vessel will be

(1) 5000 J (2) 10000 J

(3) 5 J (4) 10 J

10. A man pulls a bucket full of water from h metre deep well. If the mass of rope is m and mass of bucket full of water is M, then work done by the man is

(1) $\left(\frac{M}{2} + m\right)gh$ (2) $\left(\frac{M+m}{2}\right)gh$

(3) $\left(M + \frac{m}{2}\right)gh$ (4) $(M + m)\,gh$

11. Force constants K_1 and K_2 of two springs are in the ratio 5 : 4. They are stretched by same length. If potential energy stored in one spring is 25 J then potential energy stored in second spring is

(1) 25 J (2) 16 J

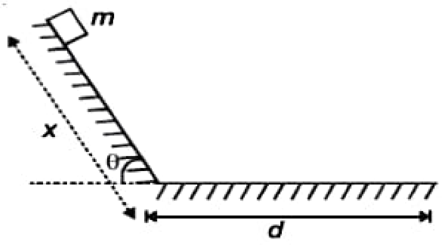
(3) 100 J (4) 20 J

12. Two springs have their force constants in the ratio of 3 : 4. Both the springs are stretched by applying equal force F. If elongation in first spring is x then elongation in second spring is

(1) $3x$ (2) $4x$

(3) $\frac{4}{3}x$ (4) $\frac{3}{4}x$

13. A block of mass m is released on the top of a smooth inclined plane of length x and inclination θ as shown in figure. Horizontal surface is rough. If block comes to rest after moving a distance d on the horizontal surface, then coefficient of friction between block and surface is



(1) $\frac{x\sin\theta}{2d}$ (2) $\frac{x\cos\theta}{2d}$

(3) $\frac{x\sin\theta}{d}$ (4) $\frac{x\cos\theta}{d}$

14. A particle of mass 3 kg is moving along x-axis and its position at time t is given by equation $x = (2t^2 + 5)$ m. Work done by all the forces acting on it in time interval $t = 0$ to $t = 3$ s is

(1) 144 J
(2) 72 J
(3) 108 J
(4) 216 J

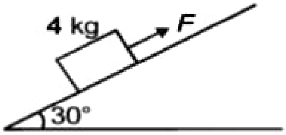
15. A force $\vec{F} = (2\hat{i} + \hat{j} + \hat{k})$ N is acting on a particle moving with constant velocity $\vec{v} = (\hat{i} + 2\hat{j} + \hat{k})$ m/s. Power delivered by force is

(1) 4 watt
(2) 5 watt
(3) 6 watt
(4) 8 watt

16. Potential energy of a particle at position x is given by $U = x^2 - 5x$. Which of the following is equilibrium position of the particle?

(1) $x = 0$
(2) $x = 5$
(3) $x = 2.5$
(4) $x = 7.5$

17. A block of mass 4 kg is pulled along a smooth inclined plane of inclination 30° with constant velocity 3 m/s as shown, power delivered by the force is



(1) 40 watt
(2) 20 watt
(3) 60 watt
(4) 30 watt

18. A stone is tied to one end of a light inextensible string of length l and made to rotate on a vertical circle keeping other end of the string at the centre. If speed of stone at the highest point is v $\left(v > \sqrt{gl}\right)$ then its speed at the lowest point is

(1) $\sqrt{4gl}$
(2) $\sqrt{v^2 + 2gl}$
(3) $\sqrt{v^2 + gl}$
(4) $\sqrt{v^2 + 4gl}$

19. Water from a stream is falling on the blades of a turbine at the rate of 100 kg/s. If the height of the stream is 100 m, then power delivered to turbine is

(1) 100 kW
(2) 0.1 kW
(3) 10 kW
(4) 1 kW

20. A body is being moved from rest along a straight line by a machine delivering constant power. The distance covered by body in time t is proportional to

(1) $\sqrt{t}$
(2) $t^{3/2}$
(3) $t^{3/4}$
(4) t^2

COLLISIONS

We shall apply the laws of conservation of momentum and energy to a common phenomenon, namely collision. (This is encountered in several games such as billiards, carrom etc.) we shall study the collision of two masses in an idealised form.

Consider two masses m_1 and m_2. The particle m_1 is moving with speed v_{1i}, the subscript i means initial, consider m_2 to be at rest. There is no loss of generality in this. This is shown in Fig. 28.

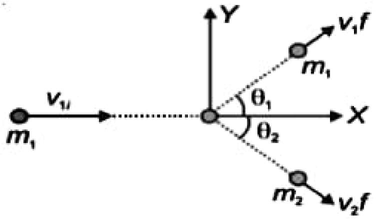


Fig. 28 : Collision of mass m_1 with stationary mass m_2

The masses fly off in different directions. Let us study the relationship between the masses, velocities and angles.

Elastic and Inelastic Collision

Total linear momentum is conserved in all collisions; the initial momentum of the system is equal to the final momentum of the system. The mutual impulsive forces acting over the collision time Δt cause a change in their respective momenta.

$$\Delta\vec{p}_1 = \vec{F}_{12}\Delta t$$

$$\Delta\vec{p}_2 = \vec{F}_{21}\Delta t$$

Where $\vec{F}_{12}$ = the force exerted on the first particle by the second particle

and $\vec{F}_{21}$ = the force exerted on the second particle by the first particle

From Newton's 3rd law, $\vec{F}_{12} = -\vec{F}_{21}$

$$\Delta\vec{p}_1 = -\Delta\vec{p}_2 \quad \text{or} \quad \Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$$

This conclusion is true even though the forces vary in complex fashion during collision time Δt. Total impulse on the first object is equal and opposite to that on the second object (as III law is true at every instant). Total KE of the system is not necessarily conserved as heat and sound may be generated during the collision. Part of the initial KE gets converted into other forms. A useful way to visualise the deformation during collision is in terms of a compressed spring'. If the 'spring' connecting the two masses regains its original shape without loss in energy, then initial KE = final KE but the KE is not constant during the collision time Δt. Such a collision is called an **elastic collision**. Deformation may not be relieved and the two bodies could move together after the collision. A collision in which the two particles move together after the collision is called a **completely inelastic collision**. The intermediate case where the deformation is partly relieved and some of the initial KE is lost is more common and is aptly called an **inelastic collision.**

Collision in One Dimension

Let us consider first a **completely inelastic collision** in one dimension. Then in Fig. 28

$$\theta_1 = \theta_2 = 0$$

Applying law of conservation of momentum,

$m_1v_{1i} + 0 = (m_1 + m_2)v_f$ (The two bodies move together with velocity v_f)

$$v_f = \frac{m_1}{m_1+m_2}v_{1i} \qquad \text{...(44)}$$

The loss in KE is

$$\Delta K = \frac{1}{2}m_1v_{1i}^2 - \frac{1}{2}(m_1+m_2)v_f^2$$

$$= \frac{1}{2}m_1v_{1i}^2 - \frac{1}{2}(m_1+m_2)\left(\frac{m_1}{m_1+m_2}v_{1i}\right)^2 \text{, Using equation (44)}$$

$$= \frac{1}{2}m_1v_{1i}^2 - \frac{1}{2}\frac{m_1^2}{m_1+m_2}v_{1i}^2$$

$$= \frac{1}{2}m_1v_{1i}^2\left[1-\frac{m_1}{m_1+m_2}\right]$$

$$= \frac{1}{2}\frac{m_1m_2}{m_1+m_2}v_{1i}^2$$

Which is a positive quantity ($m_1 > 0$, $m_2 > 0$, $v_{1i}^2 > 0$) as expected

Now consider an elastic collision using $\theta_1 = \theta_2 = 0$

Applying law of conservation of momentum

$$m_1v_{1i} = m_1v_{1f} + m_2v_{2f} \qquad \text{...(45)}$$

Where v_{1f} = velocity of body of mass m_1 after collision and v_{2f} = velocity of body of mass m_2 after collision

Applying law of conservation of kinetic energy

$$\frac{1}{2}m_1v_{1i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

or $$m_1v_{1i}^2 = m_1v_{1f}^2 + m_2v_{2f}^2 \qquad \text{...(46)}$$

Multiplying equation (45) with v_{2f} and subtracting equation (46) from it

$$m_1v_{1i} = m_1v_{1f} + m_2v_{2f} \qquad \text{...(45)} \times v_{2f}$$

$$m_1v_{1i}v_{2f} = m_1v_{1f}v_{2f} + m_2v_{2f}^2$$

$$\underset{-}{m_1v_{1i}^2} = \underset{-}{m_1v_{1f}^2} + \underset{-}{m_2v_{2f}^2} \qquad \text{...(46)}$$

$$m_1v_{1i}(v_{2f} - v_{1i}) = m_1v_{1f}(v_{2f} - v_{1f})$$

$$v_{1i}(v_{2f} - v_{1i}) = v_{1f}(v_{2f} - v_{1f})$$

$$v_{1i}v_{2f} - v_{1i}^2 = v_{1f}v_{2f} - v_{1f}^2$$

$$v_{2f}(v_{1i} - v_{1f}) = v_{1i}^2 - v_{1f}^2$$

$$v_{2f}(v_{1i} - v_{1f}) = (v_{1i} + v_{1f})(v_{1i} - v_{1f})$$

$$v_{2f} = v_{1i} + v_{1f} \qquad \text{...(47)}$$

Put this value in equation (45)

$$m_1v_{1i} = m_1v_{1f} + m_2(v_{1i} + v_{1f})$$

$$m_1v_{1i} = m_1v_{1f} + m_2v_{1i} + m_2v_{1f}$$

$$(m_1 - m_2)v_{1i} = (m_1 + m_2)v_1f$$

$$\boxed{v_{1f} = \frac{(m_1 - m_2)}{(m_1 + m_2)}v_{1i}} \qquad \ldots(48)$$

Put this value in equation (47)

$$v_{2f} = v_{1i} + \frac{m_1 - m_2}{m_1 + m_2}v_{1i}$$

$$= v_{1i}\left(1 + \frac{m_1 - m_2}{m_1 + m_2}\right)$$

$$= v_{1i}\left(\frac{m_1 + m_2 + m_1 - m_2}{m_1 + m_2}\right)$$

$$\boxed{v_{2f} = \frac{2m_1v_{1i}}{m_1 + m_2}} \qquad \ldots(49)$$

So, we have got the 'unknowns' (v_{1f} & v_{2f}) in terms of the known quantities (m_1, m_2, v_{1i})

Case-I : The two masses are equal $m_1 = m_2 = m$ (say).

$$v_{1f} = \frac{m - m}{m + m}v_{1i} \quad \text{Using equation (48)}$$

$$\boxed{v_{1f} = 0}$$

$$v_{2f} = \frac{2m\,v_{1i}}{m + m} \quad \text{Using equation (49)}$$

$$v_{2f} = v_{1i}$$

The first mass comes to rest and pushes off the second mass with its initial speed on collision.

e.g., think of striking a carrom gote with another one of same mass. First one come to rest and the second one moves with the speed of first one (which we used as a striker).

Case II : One mass dominates *e.g.*, $m_2 >> m_1$

(Put $m_2 = m$ and $m_1 = 0$ in equations (48) & (49)

$$v_{1f} = \frac{0 - m}{0 + m}v_{1i}$$

$$\boxed{v_{1f} = -v_{1i}}$$ (The higher mass reverses its velocity)

$$v_{2f} = \frac{2 \times 0}{0 + m}v_{1i}$$

$$\boxed{v_{2f} = 0}$$ (Heavier mass is undisturbed)

The heavier mass is undisturbed while the lighter mass reverses its velocity.

e.g., think of hitting a wall with a tennis ball, the ball rebounds with the same speed.

Kinetic Energy Transferred from Projectile to Target

$$K_i - K_f = \frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2$$

or $$\frac{\Delta K}{K} = \frac{\frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2}{\frac{1}{2}m_1u_1^2} = 1-\left(\frac{v_1}{u_1}\right)^2$$

Remember, $$\boxed{\frac{\Delta K}{K} = 1-\left(\frac{v_1}{u_1}\right)^2}$$

When $u_2 = 0$, $v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1$

or $$\frac{\Delta K}{K} = 1-\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$$

or $$\boxed{\frac{\Delta K}{K} = \frac{4m_1m_2}{(m_1+m_2)^2}}$$

$$= \frac{4n}{(1+n)^2}$$

if $\frac{m_1}{m_2} = n$

or $\frac{m_2}{m_1} = n$

or $$\frac{\Delta K}{K} = \frac{4m_1m_2}{(m_1-m_2)^2 + 4m_1m_2}$$

Note : (1) *Greater the difference in masses, lesser will be the transfer of K.E.*

(2) *If $m_1 = m_2$, $\frac{\Delta K}{K} = 1 = 100\%$ i.e., the K.E transfer is maximum.*

Knowledge Cloud

(1) Isolated event means "no net external force". But in practice, a collision between the colliding bodies has interactions (i.e., internal forces) much larger than the external forces acting on the system of the colliding bodies and hence external forces are neglected.

(2) Being an isolated system "the momentum is conserved in all types of collision".

i.e., $m_1\vec{u_1} + m_2\vec{u_2} = m_1\vec{v_1} + m_2\vec{v_2}$

(3) In collision "total energy" (i.e., all forms of energy such as mechanical , internal, sound, light or even mass energy) is always conserved.

(4) In a collision, the colliding bodies may or may not come in real physical touch. For example, in a collision between a ball and a bat, there is physical contact while in the collision of α-particle by a nucleus in Rutherford experiment, there is no physical touch.

Example 21 : *In a nuclear reactor, a neutron of high speed (~ 10^7 ms^{-1}) has to be slowed down to 10^3 ms^{-1} so that it can have a high probability of interacting with the isotope $^{235}_{95}U$ and cause it to fission. Show that fractional KE lost is about 10% when a neutron has an elastic collision with a high nuclei of deuterium. (The material making up the light nuclei, usually heavy water (D_2O) or graphite, is called nuclei moderator).*

Solution : Initial KE of the neutron is $K_i = \frac{1}{2}m_1v_{1i}^2$

Final KE K_{1f} using equation (48) is

$$K_{1f} = \frac{1}{2}m_1v_{1f}^2 = \frac{1}{2}m_1\left(\frac{m_1-m_2}{m_1+m_2}\right)^2 v_{1i}^2$$

Fractional KE retained is

$$f_1 = \frac{K_{1f}}{K_{1i}} = \frac{\frac{1}{2}m_1\left(\frac{m_1-m_2}{m_1+m_2}\right)^2 v_{1i}^2}{\frac{1}{2}m_1v_{1i}^2} = \left(\frac{m_1-m_2}{m_1+m_2}\right)^2$$

Fractional KE gained by moderating nuclei $\frac{K_{2f}}{K_{1i}}$ is

$$f_2 = 1 - f_1 \text{(elastic collision)}$$

$$= 1-\left(\frac{m_1-m_2}{m_1+m_2}\right)^2$$

$$= \frac{(m_1+m_2)^2-(m_1-m_2)^2}{(m_1+m_2)^2}$$

$$= \frac{4m_1m_2}{(m_1+m_2)^2}$$

You can verify this result by putting from equation (49)

For deuterium, $m_2 = 2m_1$

$$f_1 = \left(\frac{m_1-2m_1}{m_1+2m_1}\right)^2 = \frac{1}{9}$$

and $f_2 = \frac{8}{9}$

f_1 expressed as percent = $\frac{1}{9}\times 100 = 11.1\%$

or almost 90% of the neutron's energy is transferred to deuterium.

Try Yourself

42. In example 21, consider an elastic collision between a neutron and a light nuclei like carbon and calculate fractional KE lost.

Hint : $f_1 = \dfrac{4m_1m_2}{(m_1+m_2)^2}$ using the result of solved example 21

$m_2 = 12m_1$

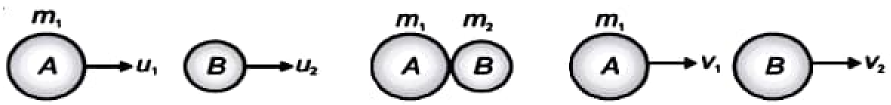
43. Consider an elastic collision between a neutron and a light nuclei like Beryllium and calculate neutron's energy transferred to Beryllium.

Hint : $f_2 = \dfrac{4m_1m_2}{(m_1+m_2)^2}$ using the result of example 21.

Use the fact that $m_2 = 9m_1$

Example 22 : *Consider objects of masses m_1 and m_2 moving initially along the same straight line with velocities u_1 and u_2 respectively. Considering a perfectly elastic collision (with $u_1 > u_2$) derive expressions for their velocities after collision.*

Solution :



Before collision During collision After collision

With $u_1 > u_2$

Relative velocity of approach before collision = $u_1 - u_2$ ...(i)

Let the velocity of *A* after collision be v_1 and the velocity of *B* after collision be v_2.

When $v_2 > v_1$, the bodies separate after collision.

Relative velocity of separation after collision = $v_2 - v_1$ ...(ii)

Linear momentum of the two balls before collision = $m_1u_1 + m_2u_2$

Linear momentum of the two balls after collision = $m_1v_1 + m_2v_2$

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2 \quad \text{...(iii)}$$

According to the law of conservation of linear momentum

or $m_2(v_2 - u_2) = m_1(u_1 - v_1)$ ...(iv)

Total KE of the two balls before collision = $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$ ...(v)

Total KE of the two balls after collision = $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$ ...(vi)

$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$ According to the law of conservation of KE

$$\frac{1}{2}m_2(v_2^2 - u_2^2) = \frac{1}{2}m_1(u_1^2 - v_1^2)$$

or $m_2(v_2^2 - u_2^2) = m_1(u_1^2 - v_1^2)$ ...(vii)

Dividing equation (vii) by equation (iv),

$$\frac{m_2(v_2^2-u_2^2)}{m_2(v_2-u_2)}=\frac{m_1(u_1^2-v_1^2)}{m_1(u_1-v_1)}$$

$$\frac{(v_2+u_2)(v_2-u_2)}{(v_2-u_2)}=\frac{(u_1+v_1)(u_1-v_1)}{(u_1-v_1)}$$

or $v_2 + u_2 = u_1 + v_1$

or $\boxed{v_2 - v_1 = u_1 - u_2}$ ...(viii)

Hence, in one-dimensional elastic collision, relative velocity of separation after collision is equal to relative velocity of approach before collision (from equations (i) & (ii))

From equation (viii), $\dfrac{v_2-v_1}{u_1-u_2}=1$

By definition, $\dfrac{v_2-v_1}{u_1-u_2}=e=1$

Hence, coefficient of restitution/resilience of a perfectly elastic collision in one dimension is unity.

From equation (viii),

$v_2 = u_1 - u_2 + v_1$ ...(ix)

Put this value in equation (iii)

$m_1v_1 + m_2(u_1 - u_2 + v_1) = m_1u_1 + m_2u_2$

$m_1v_1 + m_2u_1 - m_2u_2 + m_2v_1 = m_1u_1 + m_2u_2$

$v_1(m_1 + m_2) = (m_1 - m_2)u_1 + 2m_2u_2$

$$\boxed{v_1=\frac{(m_1-m_2)u_1}{(m_1+m_2)}+\frac{2m_2u_2}{m_1+m_2}} \quad \text{...(x)}$$

Put this value of v_1 from equation (x) in equation (ix)

$$v_2=u_1-u_2+\frac{(m_1-m_2)u_1}{m_1+m_2}+\frac{2m_2u_2}{m_1+m_2}$$

$$=u_1\left[1+\frac{m_1-m_2}{m_1+m_2}\right]+u_2\left[\frac{2m_2}{m_1+m_2}-1\right]$$

$$=u_1\left[\frac{m_1+m_2+m_1-m_2}{m_1+m_2}\right]+u_2\left[\frac{2m_2-m_1-m_2}{m_1+m_2}\right]$$

$$\boxed{v_2=\frac{2m_1u_1}{m_1+m_2}+\frac{(m_2-m_1)u_2}{(m_1+m_2)}} \quad \text{...(xi)}$$

***Note :** You can get exp. for v_2 from equation (x) by interchanging m_1 and m_2 and u_1 and u_2. You can also get equation (x) from equation (xi).*

Try Yourself

44. For example 22, considering equal masses, get the expressions for the velocities of bodies after the collision.

    Hint : Put $m_1 = m_2 = m$ (say), in equations (x) & (xi).

45. For example 22, consider target body B initially at rest, get the expressions for velocities of bodies after the collision (a) $m_1 = m_2$ (b) $m_2 >> m_1$ (c) $m_2 << m_1$

    Hint : (a) Put $m_1 = m_2$ and $u_2 = 0$ in equations (x) & (xi)

    (b) Put $m_1 = 0$ in equations (x) & (xi)

    (c) Put $m_2 = 0$ in equations (x) & (xi)

Elastic collision in two dimensions

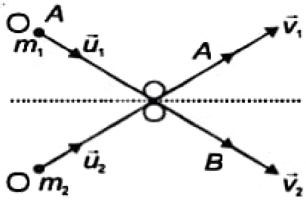
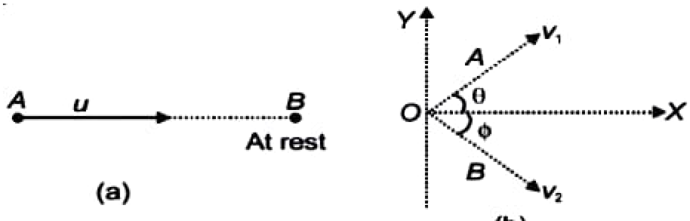


Fig. : 29

Consider two bodies moving in a plane. If they collide, and still continue moving in the same plane, the collision, is referred to as a two-dimensional (or an oblique) collision *e.g.* a collision between two billiards balls. Consider a system of two particles *A* and *B* moving, as shown fig. 29 before and after the collision. The forces, they exert on each other (during collision) are **internal forces**, and there are no other forces (*i.e.*, there is no external force), so the **linear momentum of the system is conserved. When the collision is elastic, the kinetic energy before collision is equal to kinetic energy after collision**. Consider a simple two-dimensional elastic collision, taking place, as shown below.



(a)

(b)

Fig. : 30

Figure 30(a) shows a particle *A*, of mass m_1, moving along *X*-axis, in *XY*-plane, with an initial speed *u*. Particle *B*, of mass m_2, is initially at rest. When particle *A* collides with *B*, the two particles move with speed v_1 and v_2 in *XY*-plane, after the collision (Figure 30(b)). After the collision, let *A* move in a direction inclined at an angle φ, with its initial direction of motion. Angle θ is known as the **angle of scattering.**

After the collision, let *B* move in a direction, inclined at an angle φ with the initial direction of motion of *A*. Angle θ is known as the **angle of recoil**. Knowing m_1, m_2 and *u*, we have to do the needed calculations. Applying the law of conservation of linear momentum, used for *X* and *Y* components separately, we get

$$m_1u + 0 = m_1v_1\cos\theta + m_2v_2\cos\phi \quad \text{...(50) (along X-axis)}$$

and $$0 = m_1v_1\sin\theta - m_2v_2\sin\phi \quad \text{...(51) (along Y-axis)}$$

Assuming the collision to be **perfectly elastic**,

Total KE before collision = Toal KE after collision

$$\frac{1}{2}m_1u^2 + 0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \text{(Energy is scalar, so components are not taken)}$$

or $$m_1u^2 = m_1v_1^2 + m_2v_2^2 \qquad \text{...(52)}$$

We now have three equations in all and four unknown parameters. So, a **complete solution** is **not possible**. If value of any one of the four unknowns *i.e.*, v_1, v_2, θ or ϕ, is **given**, we can calculate the remaining three using the equations (50), (51) & (52). For an **inelastic** two-dimensional collision, we have two equations *i.e.* equations (50) & (51). So in this case, if two of the four unknowns, say θ and ϕ, are given, remaining two unknowns (*i.e.*, v_1 and v_2) can be calculated using equations (50) & (51).

Special case of equal mass : Two-dimensional collision of two particles of equal mass. Equations (50), (51) & (52) reduce to

$$u = v_1\cos\theta + v_2\cos\phi \qquad \text{...(53)}$$

$$0 = v_1\sin\theta - v_2\sin\phi \qquad \text{...(54)}$$

$$u^2 = v_1^2 + v_2^2 \qquad \text{...(55)}$$

From equations (53) & (55)

$$v_1^2 + v_2^2 = (v_1\cos\theta + v_2\cos\phi)^2$$

$$v_1^2 + v_2^2 = v_1^2\cos^2\theta + v_2^2\cos^2\phi + 2v_1v_2\cos\theta\cos\phi$$

$$2v_1v_2\cos\theta\cos\phi = v_1^2(1-\cos^2\theta) + v_2^2(1-\cos^2\phi)$$

$$2v_1v_2\cos\theta\cos\phi = v_1^2\sin^2\theta + v_2^2\sin^2\phi \qquad \text{...(56)}$$

Using equation (54), equation (56) can be rewritten as

$$2v_1v_2\cos\theta\cos\phi = 2v_1^2\sin^2\theta$$

or $$\cos\theta = \left(\frac{v_1}{v_2}\right)\frac{\sin^2\theta}{\cos\phi} \qquad \text{...(57)}$$

$$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$

$$= \left(\frac{v_1}{v_2}\right)\frac{\sin^2\theta}{\cos\phi}\cos\phi - \frac{v_1}{v_2}\sin^2\theta \qquad \text{Using equations (54) \& (57)}$$

$$= \left(\frac{v_1}{v_2}\right)\sin^2\theta - \frac{v_1}{v_2}\sin^2\theta$$

$$= 0$$

$$\Rightarrow \quad \boxed{\theta + \phi = 90^\circ}$$

In the special case of a perfectly elastic (two-dimensional) collision, between two particles, of the same mass, the two particles move along mutually perpendicular directions after the collision. This is shown in the Fig. 31.

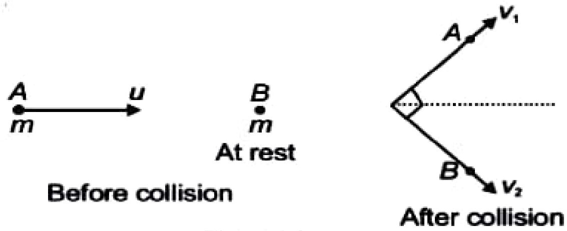
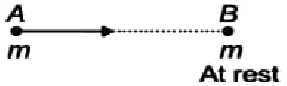


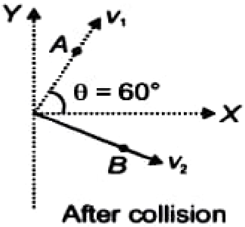
Fig. : 31

Example 23 : *Two particles A and B have the same mass m. A is moving along X-axis with a speed of 5 ms^{-1}, and B is at rest. After undergoing a perfectly elastic collision with B, particle A gets scattered through an angle of 60°. What is the direction of motion of B, and the speeds of A and B, after the collision?*

Solution :



Before collision



After collision

A & *B* have same mass and the collision is perfectly elastic

$$\therefore \quad \theta + \phi = 90°$$

$$\phi = 90° - 60° = 30° \qquad \ldots\text{(i)}$$

Using law of conservation of linear momentum

(a) For *X*-components

$$u = 5 = v_1\cos 60° + v_2\cos 30°$$

$$5 = \frac{v_1}{2} + v_2\frac{\sqrt{3}}{2}$$

$$10 = v_1 + v_2\sqrt{3} \qquad \ldots\text{(ii)}$$

(b) For *Y*-components

$$0 = v_1\sin 60° - v_2\sin 30°$$

$$v_1\frac{\sqrt{3}}{2} = \frac{v_2}{2}$$

$$v_1\sqrt{3} = v_2 \qquad \ldots\text{(iii)}$$

From equations (ii) & (iii),

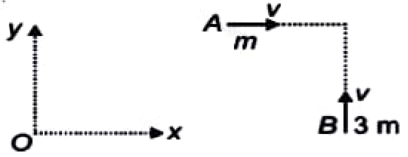
$$10 = v_1 + 3v_1 \quad \Rightarrow \quad \boxed{v_1 = 2.5\ \text{ms}^{-1}}$$

$$v_2 = v_1\sqrt{3} = 2.5 \times 1.732 \quad \Rightarrow \quad \boxed{v_2 = 4.33\ \text{ms}^{-1}}$$

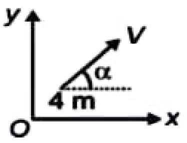
Try Yourself

46. Two particles, *A* and *B*, of masses *m* and 3*m*, are moving along *X* and *Y* axes respectively, with the same speed *v*. They collide at the origin, and coalesce into one body, after the collision. What is the velocity of this coalesced mass?

Hint :



Before collision



After collision

Let V be the speed of the combined mass and let $\vec{V}$ make an angle α with +ve x-axis

$mv = 4mV\cos\alpha$ ...(i) (conservation of momentum for x component)

$3mv = 4mV\sin\alpha$ ...(ii)

Find $\tan\alpha$,

Find $\cos\alpha$ and $\sin\alpha$ from (i) & (ii) and put in $\cos^2\alpha + \sin^2\alpha = 1$

This will give V in terms of v.

47. In Q. 46, find the loss of energy during the collision.

Hint : k_1 = Total KE before collision = $\frac{1}{2}mv^2 + \frac{1}{2}(3m)v^2$

k_2 = Total KE after collision = $\frac{1}{2}(4m)V^2$

Put the value of V found in Q. 46

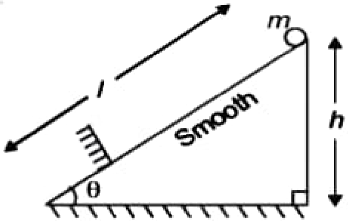
Δk = loss of KE = $k_1 - k_2$

EXERCISE

21. In perfectly elastic collision between two masses m_1 and m_2 in one dimension energy transfer is a maximum, when

(1) $m_1 = 2m_2$
(2) $m_1 << m_2$
(3) $m_1 >> m_2$
(4) $m_1 = m_2$

22. A ball of mass m is released from the top of an inclined plane of inclination θ as shown. It strikes a rigid surface at a distances $\frac{3l}{4}$ from top elastically. Impulse imparted to ball by the rigid surface is



(1) $m\sqrt{\frac{3}{2}gh}$
(2) $m\sqrt{3gh}$
(3) $2m\sqrt{3gh}$
(4) $m\sqrt{6gh}$

23. Two identical balls each of mass 4 kg are moving towards each other with speeds 2 m/s and 3 m/s respectively. They undergo head on perfectly elastic collision. Then impulse imparted by one ball on other is

(1) 12 Ns
(2) 8 Ns
(3) 20 Ns
(4) 40 Ns

24. A ball of mass 4 kg moving on a smooth horizontal surface makes an elastic collision with another ball of mass *m* at rest in the line of motion of first ball. If after collision first ball moves in the same direction with one fourth of its velocity before collision, then mass of second ball is

(1) 4 kg (2) 4.4 kg

(3) 2.4 kg (4) 2 kg

25. A heavy vehicle moving with velocity 15 m/s strikes an object of very small mass at rest head on elastically. Velocity of object after collision is

(1) 15 m/s (2) 25 m/s

(3) 20 m/s (4) 30 m/s

26. Two balls of equal masses *m* each undergo oblique collision. If collision is perfectly elastic, then angle between their velocities after collision is

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$

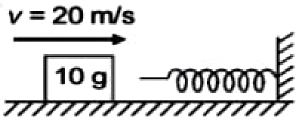
(3) $\frac{\pi}{6}$ (4) $\frac{\pi}{2}$

27. A ball falls from a height such that it strikes the floor of lift at 10 m/s. If lift is moving in the upward direction with a velocity 1 m/s, then velocity with which the ball rebounds after elastic collision will be

(1) 11 m/s (2) 12 m/s

(3) 13 m/s (4) 9 m/s

28. A block of 10 g slides on smooth horizontal surface with 20 m/s towards a spring of spring constant 100 N/m placed horizontally (as shown in figure). The maximum compression in spring is



(1) 2 m (2) 0.2 m

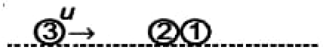
(3) 20 m (4) $\frac{4}{3}$ m

29. Two steel balls *A* and *B* of mass 10 kg and 10 g rolls towards each other with 5 m/s and 1 m/s respectively on a smooth floor. After collision; with what speed *B* moves [perfectly elastic collision]?

(1) 8 m/s (2) 10 m/s

(3) 11 m/s (4) Zero

30. Two ivory balls are placed together at rest. A third identical ball moving with velocity *u* in line of first two balls; as shown in figure collide head on elastically then



(1) Third ball comes to rest with second ball while first ball moves with speed *u*

(2) Third ball comes to rest and other two move together with speed $\frac{u}{2}$

(3) All three ball move together with speed $\frac{u}{3}$

(4) All three ball move in such a manner each makes angle 120° to each other

Inelastic Collision

If the maximum deformation in colliding bodies during deforming period is partially (not totally) recovered during recovery period, the collision is said to be "inelastic." That is

(i) During recovery period elastic potential energy of the colliding bodies decreases but remains greater than the elastic potential energy before collision and hence the loss in kinetic energy during deforming period is not recovered fully. So, **the kinetic energy after collision remains less than the kinetic energy before collision.**

(ii) During recovery period the speed of the front body increases and that of the rear body decreases. Hence the final velocity of the front body is greater than that of the rear body. So, **two bodies separate from each other** after collision.

(iii) Velocity of separation after collision is proportional to the velocity of approach before collision

i.e., $(v_2 - v_1) \propto (u_1 - u_2)$

or $\boxed{(v_2 - v_1) = e(u_1 - u_2)}$

where e is a constant depending on the nature of the colliding bodies. This constant e is called **coefficient of restitution** for the given colliding bodies. For **inelastic collision $0 < e < 1$.** So, the velocity of separation is always less than that of approach. But for elastic collision, $v_2 - v_1 = u_1 - u_2$.

It means **$e = 1$, for elastic collision.** Further we will see later that **$e = 0$ for perfectly inelastic collision.**

(iv) Solving the following equations

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

and $v_2 - v_1 = e(u_1 - u_2)$

we get, $v_1 = \dfrac{m_1 - em_2}{m_1 + m_2}u_1 + \dfrac{(1+e)m_2}{m_1 + m_2}u_2$

and $v_2 = \dfrac{m_2 - em_1}{m_1 + m_2}u_2 + \dfrac{(1+e)m_1}{m_1 + m_2}u_1$

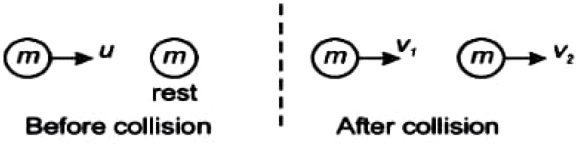
If you put $e = 1$, you will get the formulas of v_1 and v_2 for an elastic collision.

If you put $e = 0$, you will get the formulas of v_1 and v_2 for perfectly inelastic collision which will be discussed later.

(v) In an inelastic collision some mechanical energy is dissipated as heat, sound or other forms of energy also.

Example 24 : *An object of mass m moving with speed u collides one dimensionally with another identical object at rest. Find their velocities after collision, if coefficient of restitution of collisions is e.*

Solution : Let v_1 and v_2 be the final velocities of 1st and 2nd object respectively



By law of conservation of momentum $mu + m \times 0 = mv_1 + mv_2$

$\Rightarrow \quad v_1 + v_2 = u \qquad \ldots(i)$

Now,

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$\Rightarrow \quad \frac{v_2 - v_1}{u} = e$$

$$\Rightarrow \quad v_2 - v_1 = eu \qquad \ldots\text{(ii)}$$

Adding (i) and (ii)

$$2v_2 = u + eu$$

$$\Rightarrow \quad v_2 = \left[\frac{1+e}{2}\right]u$$

Subtracting (ii) from (i)

$$2v_1 = (1 - e)u$$

$$\Rightarrow \quad v_1 = \left[\frac{1-e}{2}\right]u$$

Example 25 : *A body of mass 'm' falls from height 'h' on the ground. If 'e' be the coefficient of restitution between the body and the ground, find (i) the velocity with which it rises after the nth collision with the ground (ii) The height upto which it rises after the nth collision.*

Solution : (i) Let the velocity with which the body falls on the ground is u_1, then $u_1 = \sqrt{2gh}$

The velocity with which the body rebounds after collision is v_1, then

velocity of separation = e × velocity of approach

or $\vec{v_1} = -e\vec{u_1}$ or $\boxed{v_1 = eu_1}$ (Negative sign indicates upward direction)

After rising to maximum height, the body will hit the ground with speed $v_1 = eu_1$. Hence the speed with which the body will rise will be $\boxed{v_2 = ev_1 = e^2u_1}$

Similarly, $\boxed{v_3 = ev_2 = e^3u_1}$

Proceeding similarly, the speed with which the body will rise after n^{th} collision is $\boxed{v_n = e^nu_1}$

(ii) The K.E of the body just before hitting the ground first time is $K = \frac{1}{2}mu_1^2 = mgh$

If h_1 be the height upto which the body rises after first collision then

$$mgh_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}m(eu_1)^2 = e^2 \cdot \frac{1}{2}mu_1^2 = e^2 \cdot mgh$$

or $\boxed{h_1 = e^2h}$

or $h_2 = e^2h_1$

or $\boxed{h_2 = e^4h}$

Similarly, $\boxed{h_n = e^{2n}h}$

Try Yourself

48. A ball is dropped on the ground from a height 10 m. If coefficient of restitution is 0.4, then find the height to which ball will rebound.

Perfectly Inelastic Collision

"If the maximum deformation in colliding bodies during the deforming period is not recovered at all, the collision is said to be perfectly inelastic." That is

(i) Recovery period is infinite.

(ii) The loss in K.E. during deforming period is not recovered at all. So, loss in K.E. is maximum

(iii) Since at the maximum deformation, the velocities of two colliding bodies are same and the bodies retain their maximum deformation, therefore, two bodies **do not separate** from each other after collision (*i.e.*, **they stick together**).

$$\therefore \quad e = \frac{\text{Velocity of separation after collision}}{\text{Velocity of approach before collision}}$$

$$= \frac{0}{u_1 - u_2} = 0$$

Remember **$e = 0$ for perfectly inelastic collision**

(iv) Since two colliding bodies stick together.

$\therefore$ The velocities $v_1 = v_2 = v$ (say)

Hence from conservation of momentum

$m_1u_1 + m_2u_2 = m_1v + m_2v$ (during head -on collision in one dimension)

or $$\boxed{v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}}$$

This formula is also obtained by putting $e = 0$ in the formula for v_1 and v_2 after inelastic collision given in note (iv) of the discussion of inelastic collision.

(v) The loss in K.E is

$$\Delta K = K_i - K_f = \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \frac{1}{2}(m_1 + m_2)v^2$$

Putting $v = \dfrac{m_1u_1 + m_2u_2}{m_1 + m_2}$ we get

$$\Delta K = \frac{m_1m_2(u_1 - u_2)^2}{2(m_1 + m_2)}$$

Same result is obtained by putting $e = 0$ in the formula of ΔK given in note (v) of inelastic collision.

Since $\Delta K > 0$, there is always loss in kinetic energy after a perfectly inelastic collision.

(vi) If $m_1u_1 + m_2u_2 = 0$, $v = 0$ *i.e.*, two bodies after perfectly inelastic collision come to rest if the net momentum before collision is zero.

In this case **the loss in K.E is 100%.**

(vii) In a perfectly inelastic collision, some mechanical energy is dissipated as heat, sound or other forms of energy also.

(viii) If the target is massive (*i.e.*, $m_2 >> m_1$) and $u_2 = 0$

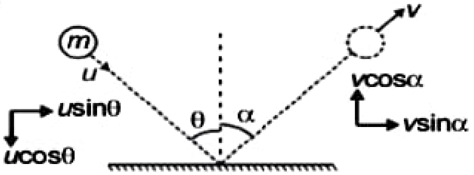
$$\Delta K = \frac{m_1 m_2}{2(m_1 + m_2)} u_1^2 \simeq \frac{1}{2} m_1 u_1^2$$

or $$\frac{\Delta K}{K} = \frac{\frac{1}{2} m_1 u_1^2}{\frac{1}{2} m_1 u_1^2} = 1 = 100\%$$

i.e., if a light moving body strikes a heavy target at rest and sticks to it, practically all its K.E is lost.

***Remark :** Some authors write "perfectly inelastic collision" as simply "inelastic collision." They write "semielastic" or "partially elastic" for "inelastic collision".*

Inelastic Collision in Two Dimensions



(Inelastic collision)

If coefficient of restitution = e

$u\sin\theta = v\sin\alpha$ ...(1)

$eu\cos\theta = v\cos\alpha$ ...(2)

$$\Rightarrow \quad v = u\sqrt{\sin^2\theta + e^2\cos^2\theta}$$

$$\tan\alpha = \frac{\tan\theta}{e}$$

Superelastic Collision

There are also cases in which the final kinetic energy is **greater** than the initial value. This occurs for example, when an explosive between the two bodies is detonated during the collision, blowing them apart with great speed. Such a collision might be called superelastic, although this term is not in common use.

Example 26 : *A particle of mass m moving with speed v in positive x-direction collides perfectly inelastically with another identical particle moving with same speed in positive y-direction. Find final velocity of the combination.*

Solution : Given, $\vec{v}_1 = v\hat{i}$ and $\vec{v}_2 = v\hat{j}$.

Let final velocity is $\vec{v}\,'$. Therefore by law of momentum conservation

$$m\vec{v}_1 + m\vec{v}_2 = 2m\vec{v}\,'$$

$$\Rightarrow \quad mv\hat{i} + mv\hat{j} = 2m\vec{v}\,'$$

$$\Rightarrow \quad \vec{v}\,' = \frac{v}{2}[\hat{i} + \hat{j}]$$

Example 27 : *A particle of mass m moving with speed u collides perfectly inelastically with another particle of mass 2m at rest. Find loss of kinetic energy of system in the collision.*

Solution : Let velocity of system of particles after collision is v.

By law of momentum conservation,

$$mu = 3mv \Rightarrow v = \frac{u}{3}$$

Now, loss of kinetic energy = $K_i - K_f$

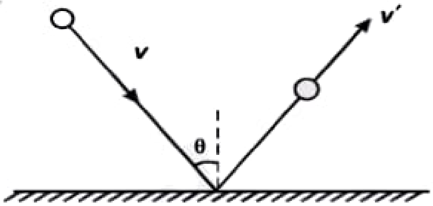
$$\Rightarrow \quad \Delta K = \frac{1}{2}mu^2 - \frac{1}{2}3m\left[\frac{u}{3}\right]^2$$

$$\Rightarrow \quad \Delta K = \frac{1}{2}mu^2 - \frac{1}{6}mu^2$$

$$\Rightarrow \quad \Delta K = \frac{1}{3}mu^2$$

Try Yourself

49. A ball moving with speed v collides with a horizontal smooth surface at an angle θ with normal to surface as shown in figure. If coefficient of restitution of collision is e, then find v'.



50. Two bodies having mass m_1 and m_2 and velocities $\vec{u}_1$ and $\vec{u}_2$ collides to form a composite system. If initial momentum of system is zero, then find the velocity of composite system..

51. An object of mass m_1 moving with speed v collides with another object of mass m_2 at rest and stick to it. Find the impulse imparted to the second object.

EXERCISE

31. A ball is dropped from height h on a horizontal floor. If it loses 60% of its energy on hitting the floor then height upto which it will rise after first rebounce is

(1) $\frac{h}{5}$

(2) $\frac{2h}{5}$

(3) $\frac{3h}{5}$

(4) $\frac{4h}{5}$

32. Two cars of same mass are moving with velocities v_1 and v_2 respectively. If they are stopped by supplying same breaking power in time t_1 and t_2 respectively then $\frac{v_1}{v_2}$ is

(1) $\frac{t_1}{t_2}$ (2) $\frac{t_1^2}{t_2^2}$

(3) $m.\frac{t_1}{t_2}$ (4) $\sqrt{\frac{t_1}{t_2}}$

33. A ball of mass m is dropped from height h on a horizontal floor which collides with it with speed u. If coefficient of restitution in floor is e, then impulse imparted to ball by the floor on its second rebounce is

(1) meu

(2) $meu (e + 1)$

(3) $me^2u (e + 1)$

(4) $me^2u (e - 1)$

34. Two identical balls moving in opposite directions with speeds 20 m/s and 25 m/s undergo head on perfectly inelastic collision. The speed of combined mass after collision is

(1) 20 m/s

(2) 22.5 m/s

(3) 25 m/s

(4) 2.5 m/s

35. A ball of mass 5 kg moving with speed 8 m/s collides head on with another stationary ball of mass 15 kg. If collision is perfectly inelastic, then loss in kinetic energy is

(1) 160 J (2) 80 J

(3) 40 J (4) 120 J

36. A ball dropped from height h on a horizontal floor goes up to the height $\frac{3h}{4}$ after hitting the floor. Fraction of energy of ball lost in the impact is

(1) $\frac{3}{4}$ (2) $\frac{1}{2}$

(3) $\frac{3}{5}$ (4) $\frac{1}{4}$

37. Two identical balls each moving with speed v at right angle to each other collide perfectly inelastically. Their speed after collision is

(1) v (2) $\sqrt{2}\,v$

(3) $\frac{v}{2}$ (4) $\frac{v}{\sqrt{2}}$

38. A ball is dropped on a horizontal surface from height h. If it rebounds upto height $\frac{h}{2}$ after first collision then coefficient of restitution between ball and surface is

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$

(3) $\frac{1}{4}$ (4) $\frac{1}{2\sqrt{2}}$

39. A body of mass m moving with a constant velocity collides head on with another stationary body of same mass. If the coefficient of restitution between the bodies is $\frac{1}{2}$ then ratio of velocities of two bodies after collision will be

(1) $\frac{1}{3}$

(2) $\frac{1}{2}$

(3) $\frac{1}{4}$

(4) 1

40. A bullet of mass m passes through a pendulum bob of mass M with velocity v and comes out of it with a velocity $\frac{v}{2}$. The minimum value of v so that the bob completes one revolution in the vertical circle is (l = length of pendulum)

(1) $\frac{M}{m}\sqrt{2gl}$

(2) $\frac{2M}{m}\sqrt{2lg}$

(3) $\frac{2M}{m}\sqrt{5lg}$

(4) $\frac{M}{m}\sqrt{5lg}$

Some Important Definitions

- The work done by the force is defined to be the product of component of the force in the direction of the displacement and the magnitude of this displacement ($W = (F\cos\theta)d = \vec{F}\cdot\vec{d}$) where θ is the angle between the direction of force $\vec{F}$ and displacement $\vec{d}$ and W is the work done).
- Energy is defined as the capacity to do work.
- Kinetic energy of a body is defined as the energy possessed due to its motion.
- Potential energy of a body is defined as the energy possessed due to its position or configuration.
- A force is conservative if (i) work done by it on an object is path independent and depends only on the end points (x_i, x_j), or (ii) the work done by the force is zero for an arbitrary closed path taken by the object, such that it returns to its initial position (iii) it can be derived from a scalar quantity U_x by the relation $\Delta v = -F(x)\Delta x$.
- The total mechanical energy of a system is conserved if the forces, doing work on it, are conservative (This is the principle of conservation of mechanical energy)
- A force is non-conservative if (i) work done by, or against the force, in moving from one point *A*, in space, to another point *B* depends on the path followed in moving from *A* to *B* or (ii) net work done by the force is not zero for an arbitrary closed path taken by the object, such that it returns to its initial position.
- The work-energy theorem states that the change in kinetic energy of a body is the work done by the net force on the body. $K_f - K_i = W_{net}$.
- For a conservative force in one dimension, **potential energy** function U_x is defined such that

$$F(x) = -\frac{dV(c)}{dx}$$

or $$U_i - U_f = \int_{x_i}^{x_f} F(x)dx$$

- The principle of conservation of energy states that the total energy of an isolated system does not change, as long as one accounts for all forms of energy or energy may be transformed from one form to another but the total energy of an isolated system remains constant.
- Power is defined as the time rate at which work is done or energy is transferred.
- The average power of a force is defined as the ratio of the work W, to the total time t taken,

  $$P_{av} = \frac{W}{t}$$

- The instantaneous power is defined as the limiting value of the average power as time interval approaches zero, $P = \frac{dW}{dt}$. It can also be expressed as $P = \vec{F}\cdot\frac{d\vec{r}}{dt} = \vec{F}\cdot\vec{v}$ where $\vec{v}$ is the instantaneous velocity when the force in $\vec{F}$ & $\vec{r}$ is the displacement vector.
- An **elastic collision** in the one is which the initial kinetic energy is equal to the final kinetic energy (but the *KE* during the collision time Δt is not constant).
- A **completely elastic collision** is the one in which the two particles move together after the collision.
- An **inelastic collision** is the one in which deformation is partly relieved and some of the initial kinetic energy is lost.

Formulae Chart

1. The **scalar or dot product** of two vectors $\vec{A}$ and $\vec{B}$ is $\vec{A}\cdot\vec{B}$ given by $\vec{A}\cdot\vec{B} = AB\cos\theta$, where θ is the angle between $\vec{A}$ and $\vec{B}$. It is the product of magnitude of one vector and component of the other vector along the first vector. For unit vector:

   $$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1 \text{ and } \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$$

2. The **work** done by a force $\vec{F}$ acting at an angle θ with the direction of displacement $\vec{d}$ is $W = (F\cos\theta)d$.
3. The **work-energy theorem** states that the change in kinetic energy of a body is the net work done by the net force on the body.

   $$K_f - K_i = W_{net}$$

   Where K_i and K_f are initial and final kinetic energies and W_{net} is the net work done.
4. For a conservative force in one dimension, **potential energy function** $V(x)$ is defined such that

   $$F(x) = -\frac{dV(x)}{dx}$$

   or $$V_i - V_f = \int_{x_i}^{x_f} F(x)dx$$

5. The **kinetic energy**, $K = \frac{1}{2}m\vec{v}.\vec{v} = \frac{1}{2}mv^2$ where m is mass of the body moving with velocity $\vec{v}$.

6. The **gravitational potential energy** of a particle of mass m at a height x above the Earth's surface is

   $V(x) = mgx$

   Variation of g with height is ignored.

7. The elastic potential energy of a spring of force constant k and extension x is $V(x) = \frac{1}{2}kx^2$.

8. Average power of a force is defined as the ratio of the work, W, to the total time t taken.

   $$P_{av} = \frac{W}{t}$$

9. The instantaneous power is defined as the limiting value of the average power as time interval approaches zero,

   $$P = \frac{dW}{dt}$$

   It can also be expressed as $P = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$, where $d\vec{r}$ is displacement vector

   For equations (10) to (18), *i.e.*, for objects of masses m_1 and m_2 moving initially along the same straight line with velocities u_1 and u_2 and $u_1 > u_2$ and velocity v_1 and v_2 after the collision.

   For perfectly elastic collision

10. Relative velocity of approach before collision = $u_1 - u_2$
11. Relative velocity of separation after collision = $v_2 - v_1$
12. $v_2 - v_1 = u_1 - u_2$
13. $v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} + \frac{2m_2u_2}{m_1 + m_2}$
14. $v_2 = \frac{2m_1u_1}{m_1 + m_2} + \frac{(m_2 - m_1)}{m_1 + m_2}u_2$
15. For equal masses, $v_1 = u_2$ and $v_2 = u_1$
16. For equal masses, and target body at rest

    $v_1 = 0$ and $v_2 = u_1$
17. For target body at rest and $m_2 >> m_1$

    $v_1 = -u_1$ and $v_2 = 0$
18. For target body at rest and $m_2 << m_1$

    $v_1 = u_1$ and $v_2 = 2u_1$
19. For **elastic collision** in **one dimension**.

    A particle of mass m_1 moving along X-axis with initial speed u collides with particle B of mass m_2 at rest and after the collision A moves in a direction inclined at an angle θ with its initial direction of motion and B moves in a direction inclined at an angle ϕ with the initial direction of motion of A. A and B move with speed v_1 & v_2 after the collision

    $m_1u = m_1v_1\cos\theta + m_2v_2\cos\phi$

    and $0 = m_1v_1\sin\theta - m_2v_2\sin\phi$

    and $m_1u^2 = m_1v_1^2 + m_2v_2^2$
20. For an **elastic two-dimensional collision** with bodies of equal masses

    $u = v_1\cos\theta + v_2\cos\phi$, $0 = v_1\sin\theta - v_2\sin\phi$

    $u^2 = v_1^2 + v_2^2$ and $\theta + \phi = 90°$

Quick Recap

1. Work done by a force is defined as the product of component of the force in the direction of the displacement and the magnitude of this displacement. $W = (F\cos\theta)d$

2. Work is a scalar quantity. It can be positive, negative or zero unlike mass and kinetic energy which are positive scalar quantities. The work done by the friction or viscous, force is negative. When we say 'calculate the work done' we need to mention the work done by a specific force or a group of forces on a given body over a certain displacement, otherwise the phrase is incomplete.

3. SI unit of work is joule (J),

   CGS unit is erg.

   $1\ \text{J} = 10^7\ \text{erg}$

   Gravitational unit of work in SI is kg m and

   Gravitational unit of work in CGS is g cm

   1 kg m = 9.8 J,

   1 g cm = 980 erg,

   $1\ \text{kg m} = 10^5\ \text{g cm}$

4. For two bodies, the sum of the mutual forces exerted between them is zero from Newton's III law, $\vec{F}_{12} + \vec{F}_{21} = 0$ but the sum of the work done by the two forces need not always cancel, *i.e.*, $W_{12} + W_{21} \neq 0$. However, it may sometimes be true.

5. The work-energy theorem states that the change in kinetic energy of a body is the work done by the net force on the body $K_f - K_i = W_{net}$

6. The W-E theorem is not independent of Newton's II law. It may be viewed as a scalar form of the II law. The principle of conservation of mechanical energy may be viewed as a consequence of the W-E theorem for conservative forces.

7. The W-E theorem holds in all inertial frames. It can also be extended to non-inertial frames provided we include the pseudoforces in the calculation of the net force acting on the body under consideration.

8. A force is conservative if (i) work done by it on an object is path independent or (ii) the work done by the force is zero for an arbitrary closed path taken by the object such that it returns to its initial position or (iii) it can be derived from a scalar quantity $V(x)$ by the relation $V(x) = -\ F(x)\Delta x$

9. For a conservative force in one dimension, a **potential energy** function $v(x)$ is defined such that

$$F(x) = -\frac{dV(x)}{dx} \quad \text{or} \quad V_i - V_f = \int_{x_i}^{x_f} F(x)dx$$

10. The principle of conservation of mechanical energy states that the total mechanical energy of a body remains constant if the only forces that act on the body are conservative.

11. The elastic potential energy of a spring of force constant k and extension x is $V(x) = \frac{1}{2}kx^2$.

12.

Physical quantity	Symbol	Dimension	Units	Remarks
Work	W	$[ML^2T^{-2}]$	J	$W=\vec{F}.\vec{d}$
Kinetic energy	K	$[ML^2T^{-2}]$	J	$K=\frac{1}{2}mv^2$
Potential energy	V(x)	$[ML^2T^{-2}]$	J	$F(x)=-\frac{dV(x)}{dx}$
Mechanical energy	E	$[ML^2T^{-2}]$	J	$E=K+V$
Spring constant	K	$[MT^{-2}]$	Nm^{-1}	$F=-kx$ $V(x)=\frac{1}{2}kx^2$
Power	P	$[ML^2T^{-3}]$	W	$P=\vec{F}\cdot\vec{v}$ $P=\frac{dW}{dt}$

13. Every force encountered in mechanics does not have an associated potential energy *e.g.*, work done by friction over a closed path is not zero and no potential energy can be associated with zero.

14. An **elastic collision** is the one in which the initial kinetic energy is equal to the final kinetic energy. A **completely inelastic collision** is the one in which the two particles move together after the collision. An **inelastic collision** is the one in which deformation is partly relieved and some of the initial kinetic energy is lost.

15. During a collision

(a) The total linear momentum is conserved at each instant of the collision.

(b) In elastic collision kinetic energy before collision is equal to kinetic energy after collision.

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Assignment

(SET - 1)

School/Board Examinations

Students are required to solve and write the solutions in their exercise book.

For referring solutions to the assignment (Set-1), please visit our Library at the Centre or log on to our website: www.aakash.ac.in

SECTION - A

School/Board Exam. Type Questions

Very Short Answer Type Questions :

1. Can KE of a system be increased without applying any external force?
2. When is the exchange of energy maximum during an elastic collision?
3. Comment on the nature of work done by the stretching force on a spring.
4. What happens to the PE of a spring as it is compressed or stretched?
5. What is the source of KE of falling rain drops?
6. Does kinetic energy depend on the direction of motion? Can it be negative?
7. Does work done in moving a body depend on how fast or how slow the body is moved?
8. Does KE of a system depend on frame of reference?
9. What happens to the PE of an air bubble as it rises in water?
10. Name the smallest and largest practical unit of energy.

Short Answer Type Questions :

11. What happens to the PE of the system as two electrons are brought towards each other? What happens when a proton and an electron are brought nearer?
12. The dot product of two vectors is zero when the vectors are orthogonal and maximum when they are parallel to each other. Explain.
13. Explain scalar product of two vectors and give its geometrical interpretation.
14. If $\vec{P}, \vec{Q}$ and $\vec{R}$ are non-zero vectors and $\vec{P}\cdot\vec{Q}=0$ and $\vec{Q}\cdot\vec{R}=0$ then find $\vec{P}\cdot\vec{R}$
15. A body moves from a point $\vec{r}_1=(2\hat{i}+3\hat{j})$ to another point $\vec{r}_2=(3\hat{i}+2\hat{j})$ due to force $\vec{F}=5\hat{i}+5\hat{j}$. Find the work done by the force.
16. A uniform chain of length l and mass m is lying on a smooth table and one-fourth of its length is hanging vertically over the edge of the table. Find the work needed to be done to pull the hanging part on to the table.
17. A man rowing a boat upstream is at rest w.r.t. the shore (i) Comment on the work done by him (ii) Is any work done by him when he stops rowing and moves along with the stream?
18. Increase in KE of a car is E_1 when it is accelerated from 15 ms^{-1} to 20 ms^{-1} and increase in KE of a car is E_2 when it is accelerated from 20 ms^{-1} to 25 ms^{-1}. Find $\frac{E_1}{E_2}$.
19. There are two springs A and B. Both have spring constant of 100 Nm^{-1}. A is compressed by 5 cm. B is stretched by 5 cm. What is the difference in PE stored in the two springs?
20. Comment on change in PE
    (a) A body is taken against the gravitational force
    (b) Air bubble rises up in water
    (c) A spring is stretched
21. Can a body have momentum without mechanical energy?

22. Can a body have momentum when its mechanical energy is negative?

23. Can a body have mechanical energy without momentum?

24. For a planet moving around the sun, do you think it is a good illustration of the law of conservation of energy?

25. Justify the statement: fast neutrons can be slowed down by passing them through heavy water.

26. (a) Two bodies of masses m_1 and m_2 ($m_2 > m_1$) are dropped from the same height. Compare their momentum just before hitting the ground.

    (b) Two bodies of masses m_1 and m_2 ($m_2 > m_1$) have same KE. Compare their momentum.

27. A body of mass 60 kg moving with velocity 2 ms^{-1} collides with a body of mass 40 kg moving with velocity 4 ms^{-1}. Calculate the loss of energy assuming perfectly inelastic collision.

28. A meteorite burns in the atmosphere before reaching Earth's surface. What happens to its momentum? Is momentum conservation principle violated?

29. What is meant by mass energy equivalence?

30. A body of mass 10 kg is dropped from a height of 15 m. Assuming that the whole of the energy is converted into heat energy, find the heat energy in calorie. Given g = 10 ms^{-2} and Joule's mechanical equivalent of heat = 4 J cal^{-1}.

Long Answer Type Questions :

31. Give examples of dot product of two vectors. Discuss briefly some of the properties of dot product.

32. Explain the term 'work'. Get an expression for work done by a constant force. What are its dimensions and SI unit?

33. Discuss the absolute and gravitational units of work on SI and CGS systems.

34. Explain the meaning of KE with examples. Get an expression for KE of a body moving uniformly.

35. What is meant by PE of a spring? Get an expression for it.

36. Discuss the variation of PE (associated with spring) with distance x. Also, discuss, variations of KE and total energy graphically.

    OR

    A small object of weight w hangs from a string of length l. A variable horizontal force, F, which starts at zero value and increases gradually is used to pull the object very slowly until the string makes an angle θ with the vertical. Prove that the work done by the force is $wl(1 - \cos\theta)$.

37. Compare the properties of conservative and non-conservative forces with regard to

    (i) PE

    (ii) Reversibility of work done

    (iii) Dependence on the path

    (iv) Work done in a round trip. Give examples

38. Consider an object of mass m connected with an ideal spring of spring constant k. Show that the speed and the KE will be maximum at equilibrium position. Get an expression for the maximum speed.

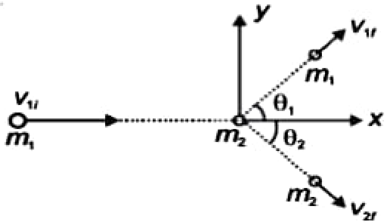
39. Illustrate the conservation of mechanical energy for vibration of a simple pendulum. Explain it with the variation of KE, PE and total energy with the position.

40. Explain 'potential energy'. Give its dimensions and SI unit. Give an example of potential energy other than that of gravitational PE.

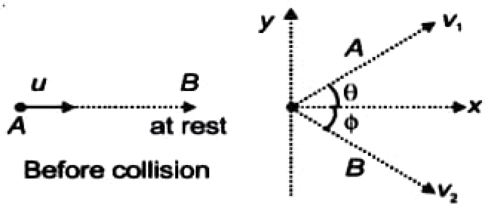
41. Mention some of the different forms of energy and discuss them briefly.

42. What do you mean by collision? Discuss two types of collision with their essential characteristics.

43. Consider the collision shown in Fig. between two billiard balls with equal masses. The player wants to 'sink' the target ball in a corner pocket, which is at an angle $\theta_2 = 30°$. Assume an elastic collision. Neglect friction and rotational motion. Find θ_1



44. Discuss elastic collision in two dimensions briefly.



After collision

45. For Q.44, take $m_1 = m_2$. Get an equation involving θ and ϕ.

OR

Consider a perfectly inelastic collision in one dimension. Show that some KE is always lost in it.

SECTION - B

Model Test Paper

Very Short Answer Type Questions : **[1 Mark]**

1. Does KE depend upon the direction of motion?

2. Is work done by a non-conservative force always negative. Comment.

3. Can PE of a body be negative?

4. Does the work done in raising a body depend upon how fast it is raised?

5. What is the work done by centripetal force?

6. How does PE of a spring change when it is compressed or stretched?

7. A light and a heavy body have same KE, which body has more momentum?

8. In which type of collision, elastic or inelastic, the momentum is conserved?

9. In which type of collision, elastic or inelastic, KE is conserved?

10. Two balls of same mass moving in opposite direction with same speed collide head-on with each other. Assuming a perfectly elastic collision, predict the outcome.

Short Answer Type Questions : [2 Marks]

11. (a) Can KE of a system be changed without changing its momentum?

    (b) Can momentum of a system be changed without changing its KE?

12. The momentum of a body is increased by 50%. Find the percentage change in KE

13. If $\vec{P}\cdot\vec{Q}=\vec{P}\cdot\vec{R}$ can we conclude that $\vec{Q}=\vec{R}$?

14. A particle moves from position $\vec{r}_1=(2\hat{i}+3\hat{j}-4\hat{k})$m to position $\vec{r}_2=(7\hat{i}+8\hat{j}+\hat{k})$m under the action of a force $\vec{F}=(2\hat{i}+2\hat{j}-2\hat{k})$ newton. Calculate the work done

15. Can a body have energy without momentum?

16. Two springs P and Q are identical but P is harder than Q ($K_P > K_Q$). On which spring, more work will be done, if

    (i) They are stretched by same force?

    (ii) They are stretched through same distance?

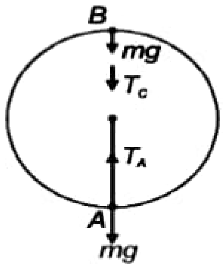
Short Answer Type Questions : [3 Marks]

17. What do you mean by zero work? State the conditions for zero work. Give one example.

18. Define work. State its dimensions, CGS & SI unit.

19. A bullet of mass 100 g moving with a velocity of 300 ms^{-1} strikes a wall and goes out of the other side with a velocity of 100 ms^{-1}. Calculate work done in passing through the wall.

20. A bob of mass m is suspended by a light string of length l. It moves with horizontal velocity v at the lowest point A such that it completes a semicircular trajectory in the vertical plane with the string becoming slack only on reaching the top-most point B. Show that minimum speed at A should be $\sqrt{5gl}$.



21. What is meant by mass-energy equivalence?

22. A ball of mass 0.1 kg makes an elastic head-on collision with a ball of unknown mass that is initially at rest. If 0.1 kg ball rebounds at one-third of its original speed, what is the mass of the other ball?

Long Answer Type Questions : **[5 Marks]**

23. Consider two bodies of masses m_1 and m_2 moving with velocities u_1 and u_2 along the same straight line with $u_1 > u_2$. Find their velocities after the collision, assuming an elastic collision.

24. State and explain work-energy principle.

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Assignment

(SET - 2)

NEET & AIIMS

(Competitive Entrance Exams.)

SECTION - A

Objective Type Questions

1. A string is used to pull a block of mass m vertically up by a distance h at a constant acceleration $\frac{g}{3}$. The work done by the tension in the string is

(1) $\frac{2}{3}mgh$ (2) $\frac{-mgh}{3}$

(3) mgh (4) $\frac{4}{3}mgh$

2. A particle moves along X-axis from $x = 0$ to $x = 1$ m under the influence of a force given by $F = 3x^2 + 2x - 10$. Work done in the process is

(1) +4 J (2) –4 J

(3) +8 J (4) –8 J

3. A body constrained to move in z direction is subjected to a force given by $\vec{F} = (3\hat{i} - 10\hat{j} + 5\hat{k})$N. What is the work done by this force in moving the body through a distance of 5 m along z-axis?

(1) 15 J

(2) – 15 J

(3) –50 J

(4) 25 J

4. If 250 J of work is done in sliding a 5 kg block up an inclined plane of height 4 m. Work done against friction is (g = 10 ms^{-2})

(1) 50 J (2) 100 J

(3) 200 J (4) Zero

5. A man carries a load on his head through a distance of 5 m. The maximum amount of work is done when he

(1) Moves it over an inclined plane

(2) Moves it over a horizontal surface

(3) Lifts it vertically upwards

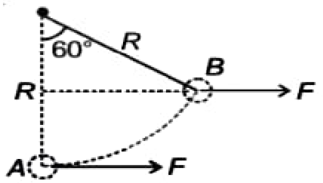
(4) None of these

6. A body moves a distance of 10 m along a straight line under the action of a force 5 N. If the work done is 25 joule, the angle which the force makes with the direction of motion of body is

(1) 0° (2) 30°

(3) 60° (4) 90°

7. A block of mass m is pulled along a circular arc by means of a constant horizontal force F as shown. Work done by this force in pulling the block from A to B is



(1) $\frac{FR}{2}$ (2) FR

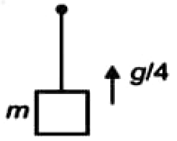
(3) $\frac{\sqrt{3}}{2}FR$ (4) mgR

8. A particle is displaced from a position $(2\hat{i} - \hat{j} + \hat{k})$ metre to another position $(3\hat{i} + 2\hat{j} - 2\hat{k})$ metre under the action of force $(2\hat{i} + \hat{j} - \hat{k})$ N. Work done by the force is

(1) 8 J (2) 10 J

(3) 12 J (4) 36 J

9. A string is used to pull a block of mass m vertically up by a distance h at a constant acceleration $\frac{g}{4}$. The work done by the tension in the string is



(1) $+\frac{3mgh}{4}$

(2) $-\frac{mgh}{4}$

(3) $+\frac{5}{4}mgh$

(4) + mgh

10. Work done by frictional force

(1) Is always negative

(2) Is always positive

(3) Is zero

(4) May be positive, negative or zero

11. Under the action of a force, a 2 kg body moves such that its position x as a function of time t is given by $x = \frac{t^2}{3}$, where x is in metre and t in second. The work done by the force in first two seconds is

(1) 1600 J (2) 160 J

(3) 16 J (4) $\frac{16}{9}$ J

12. A rifle bullets loses $\left(\frac{1}{20}\right)$th of its velocity in passing through a plank. Assuming that the plank exerts a constant retarding force, the least number of such planks required just to stop the bullet is

(1) 11 (2) 20

(3) 21 (4) Infinite

13. A particle moves along x-axis from $x = 0$ to $x = 5$ metre under the influence of a force $F = 7 - 2x + 3x^2$. The work done in the process is

(1) 70

(2) 135

(3) 270

(4) 35

14. A particle of mass 2 kg travels along a straight line with velocity $v = a\sqrt{x}$, where a is a constant. The work done by net force during the displacement of particle from $x = 0$ to $x = 4$ m is

(1) a^2 (2) $2a^2$

(3) $4a^2$ (4) $\sqrt{2}\,a^2$

15. The position x of a particle moving along x-axis at time (t) is given by the equation $t = \sqrt{x} + 2$, where x is in metres and t in seconds. Find the work done by the force in first four seconds.

(1) Zero (2) 2 J

(3) 4 J (4) 8 J

16. A uniform chain of length L and mass M is lying on a smooth table and one third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, the minimum work required to pull the hanging part of the chain on the table is

(1) MgL (2) $\frac{MgL}{3}$

(3) $\frac{MgL}{9}$ (4) $\frac{MgL}{18}$

17. Two bodies of masses m_1 and m_2 have same kinetic energy. The ratio of their momentum is

(1) $\sqrt{\frac{m_2}{m_1}}$ (2) $\sqrt{\frac{m_1}{m_2}}$

(3) $\frac{m_1^2}{m_2^2}$ (4) $\frac{m_2^2}{m_1^2}$

18. Two bodies of masses m_1 and m_2 have same momentum. The ratio of their KE is

(1) $\sqrt{\frac{m_2}{m_1}}$ (2) $\sqrt{\frac{m_1}{m_2}}$

(3) $\frac{m_1}{m_2}$ (4) $\frac{m_2}{m_1}$

19. KE of a body is increased by 44%. What is the percent increase in the momentum?

(1) 10% (2) 20%

(3) 30% (4) 44%

20. When momentum of a body increases by 200%, its KE increases by

(1) 200% (2) 300%

(3) 400% (4) 800%

21. Two bodies of masses m_1 and m_2 are moving with same kinetic energy. If P_1 and P_2 are their respective momentum, the ratio $\frac{P_1}{P_2}$ is equal to

(1) $\frac{m_1}{m_2}$ (2) $\sqrt{\frac{m_2}{m_1}}$

(3) $\sqrt{\frac{m_1}{m_2}}$ (4) $\frac{{m_1}^2}{{m_2}^2}$

22. KE acquired by a mass m in travelling a certain distance d, starting from rest, under the action of a constant force F is

(1) Directly proportional to $\sqrt{m}$

(2) Directly proportional to m

(3) Directly proportional to $\frac{1}{m}$

(4) None of these

23. A simple pendulum with bob of mass m and length x is held in position at an angle θ_1 and then angle θ_2 with the vertical. When released from these positions, speeds with which it passes the lowest positions are v_1 & v_2 respectively. Then, $\frac{v_1}{v_2}$ is

(1) $\frac{1-\cos\theta_1}{1-\cos\theta_2}$ (2) $\sqrt{\frac{1-\cos\theta_1}{1-\cos\theta_2}}$

(3) $\sqrt{\frac{2gx(1-\cos\theta_1)}{1-\cos\theta_2}}$ (4) $\sqrt{\frac{1-\cos\theta_1}{2gx(1-\cos\theta_2)}}$

24. A U–238 nucleus originally at rest, decays by emitting an α-particle, say with a velocity of v m/s. The recoil velocity (in ms^{-1}) of the residual nucleus is

(1) $\frac{4v}{238}$ (2) $-\frac{4v}{238}$

(3) $\frac{v}{4}$ (4) $-\frac{4v}{234}$

25. The total work done on a particle is equal to the change in its kinetic energy. This is applicable

(1) Always

(2) Only if the conservative forces are acting on it

(3) Only in inertial frames

(4) Only when pseudo forces are absent

26. Potential energy is defined

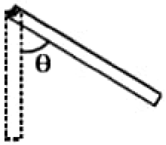
(1) Only in conservative fields

(2) As the negative of work done by conservative forces

(3) As the negative of workdone by external forces when $\Delta K = 0$

(4) All of these

27. A stick of mass m and length l is pivoted at one end and is displaced through an angle θ. The increase in potential energy is



(1) $mg\frac{l}{2}(1-\cos\theta)$

(2) $mg\frac{l}{2}(1+\cos\theta)$

(3) $mg\frac{l}{2}(1-\sin\theta)$

(4) $mg\frac{l}{2}(1+\sin\theta)$

28. A spring with spring constants k when compressed by 1 cm, the potential energy stored is U. If it is further compressed by 3 cm, then change in its potential energy is

(1) $3U$

(2) $9U$

(3) $8U$

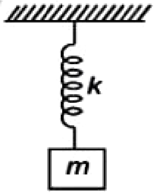
(4) $15U$

29. Two springs have force constant K_1 and K_2 ($K_1 > K_2$). Each spring is extended by same force F. It their elastic potential energy are E_1 and E_2 then $\frac{E_1}{E_2}$ is

(1) $\frac{K_1}{K_2}$ (2) $\frac{K_2}{K_1}$

(3) $\sqrt{\frac{K_1}{K_2}}$ (4) $\sqrt{\frac{K_2}{K_1}}$

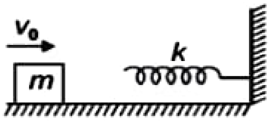
30. Initially mass m is held such that spring is in relaxed condition. If mass m is suddenly released, maximum elongation in spring will be



(1) $\frac{mg}{k}$ (2) $\frac{2mg}{k}$

(3) $\frac{mg}{2k}$ (4) $\frac{mg}{4k}$

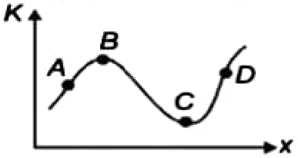
31. A block of mass m moving with velocity v_0 on a smooth horizontal surface hits the spring of constant k as shown. The maximum compression in spring is



(1) $\sqrt{\frac{2m}{k}}\cdot v_0$ (2) $\sqrt{\frac{m}{k}}\cdot v_0$

(3) $\sqrt{\frac{m}{2k}}\cdot v_0$ (4) $\frac{m}{2k}\cdot v_0$

32. For a particle moving under the action of a variable force, kinetic energy-position graph is given, then



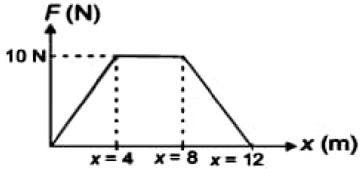
(1) At A particle is decelerating

(2) At B particle is accelerating

(3) At C particle has maximum velocity

(4) At D particle has maximum acceleration

33. A particle of mass 0.1 kg is subjected to a force which varies with distance as shown. If it starts its journey from rest at $x = 0$, then its velocity at $x = 12$ m is



(1) 0 m/s (2) $20\sqrt{2}$ m/s

(3) $20\sqrt{3}$ m/s (4) 40 m/s

34. An unloaded bus can be stopped by applying brakes on straight road after covering a distance x. Suppose, the passenger add 50% of its weight as the load and the braking force remains unchanged, how far will the bus go after the application of the brakes? (Velocity of bus in both case is same)

(1) Zero (2) $1.5x$

(3) $2x$ (4) $2.5x$

35. The potential energy of a particle of mass 5 kg moving in x-y plane is given by equation, $U = -7x + 24y$ Joule. Here x and y are in metre at $t = 0$, the particle is at origin and moving with velocity $(2\hat{i} + 3\hat{j})$ m/s. The magnitude of acceleration of particle is

(1) 3 m/s^2 (2) 5 m/s^2

(3) 31 m/s^2 (4) 15 m/s^2

36. The power of water pump is 4 kW. If $g = 10$ ms^{-2}, the amount of water it can raise in 1 minute to a height of 20 m is

(1) 100 litre (2) 1000 litre

(3) 1200 litre (4) 2000 litre

37. A particle moves with the velocity $\vec{v} = (5\hat{i} + 2\hat{j} - \hat{k})\text{ms}^{-1}$ under the influence of a constant force, $\vec{F} = (2\hat{i} + 5\hat{j} - 10\hat{k})$ N. The instantaneous power applied is

(1) 5 W (2) 10 W

(3) 20 W (4) 30 W

38. A body is projected from ground obliquely. During downward motion, power delivered by gravity to it

(1) Increases

(2) Decreases

(3) Remains constant

(4) First decreases and then becomes constant

39. The blades of a wind mill sweep out a circle of area A. If wind flows with velocity v perpendicular to blades of wind mill and its density is ρ, then the mechanical power received by wind mill is

(1) $\dfrac{\rho A v^3}{2}$ (2) $\dfrac{\rho A v^2}{2}$

(3) $\rho A v^2$ (4) $2\rho A v^2$

40. A body of mass m accelerates uniformly from rest to velocity v_1 in time interval T_1. The instantaneous power delivered to the body as a function of time t is

(1) $\dfrac{mv_1^2}{T_1^2}t$

(2) $\dfrac{mv_1}{T_1^2}t$

(3) $\left(\dfrac{mv_1}{T_1}\right)^2 t$

(4) $\dfrac{mv_1^2}{T_1}t^2$

41. The power of a pump, which can pump 500 kg of water to height 100 m in 10 s is

(1) 75 kW (2) 25 kW

(3) 50 kW (4) 500 kW

42. A pump is used to pump a liquid of density ρ continuously through a pipe of cross section area A. If liquid is flowing with speed V, then power of pump is

(1) $\dfrac{1}{3}\rho A V^2$ (2) $\dfrac{1}{2}\rho A V^2$

(3) $2\rho A V^2$ (4) $\dfrac{1}{2}\rho A V^3$

43. A car of mass m has an engine which can deliver power P. The minimum time in which car can be accelerated from rest to a speed v is

(1) $\dfrac{mv^2}{2P}$ (2) Pmv^2

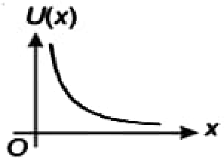
(3) $2Pmv^2$ (4) $\dfrac{mv^2}{2}P$

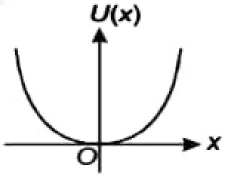
44. From a water fall, water is pouring down at the rate of 100 kg/s, on the blades of a turbine. If the height of the fall is 100 m, the power delivered to the turbine is approximately equal to

(1) 100 kW (2) 10 kW

(3) 1 kW (4) 100 W

45. On a particle placed at origin a variable force $F = -ax$ (where a is a positive constant) is applied. If $U(0) = 0$, the graph between potential energy of particle $U(x)$ and x is best represented by

(1)


(2)


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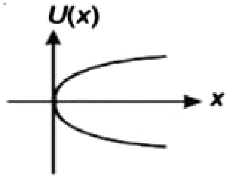

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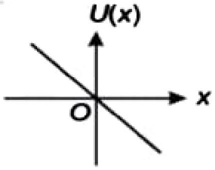
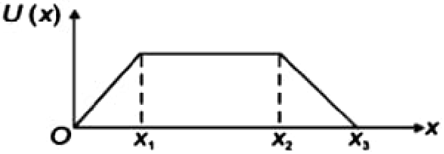
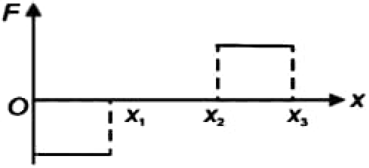
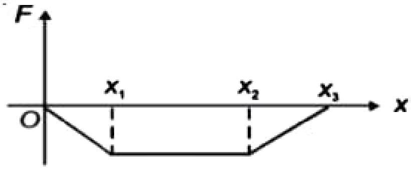
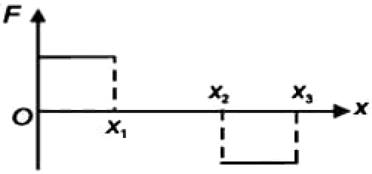


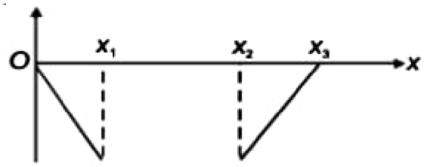
46. The variation of potential energy U of a system is shown in figure. The force acting on the system is best represented by



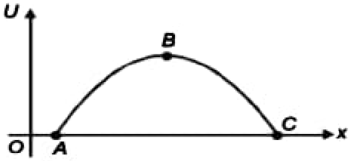
(1)


(2)


(3)


(4)


47. The variation of potential energy U of a body moving along x-axis varies with its position (x) as shown in figure



The body is in equilibrium state at

(1) *A* (2) *B*

(3) *C* (4) Both *A* & *C*

48. A particle of mass 200 g is moving in a circle of radius 2 m. The particle is just 'looping the loop'. The speed of the particle and the tension in the string at highest point of the circular path are ($g = 10$ ms^{-2})

(1) 4 ms^{-1}, 5 N (2) 4.47 ms^{-1}, zero

(3) 2.47 ms^{-1}, zero (4) 1 ms^{-1}, zero

49. A particle of mass 200 g, is whirled into a vertical circle of radius 80 cm using a massless string. The speed of the particle when the string makes an angle of 60° with the vertical line is 1.5 ms^{-1}. The tension in the string at this position is

(1) 1 N (2) 1.56 N

(3) 2 N (4) 3 N

50. A stone of mass 1 kg is tied with a string and it is whirled in a vertical circle of radius 1 m. If tension at the highest point is 14 N, then velocity at lowest point will be

(1) 3 m/s (2) 4 m/s

(3) 6 m/s (4) 8 m/s

51. An object of mass 80 kg moving with velocity 2 ms^{-1} hit by collides with another object of mass 20 kg moving with velocity 4 ms^{-1}. Find the loss of energy assuming a perfectly inelastic collision

(1) 12 J (2) 24 J

(3) 30 J (4) 32 J

52. A ball of mass m moving with velocity v collides head-on with the second ball of mass m at rest. If the coefficient of restitution is e and velocity of first ball after collision is v_1 and velocity of second ball after collision is v_2 then

(1) $v_1 = \frac{(1-e)u}{2}, v_2 = \frac{(1+e)u}{2}$

(2) $v_1 = \frac{(1+e)u}{2}, v_2 = \frac{(1-e)u}{2}$

(3) $v_1 = \frac{u}{2}, v_2 = -\frac{u}{2}$

(4) $v_1 = (1+e)u$, $v_2 = (1-e)u$

53. Particle A makes a perfectly elastic collision with another particle B at rest. They fly apart in opposite direction with equal speeds. If their masses are m_A & m_B respectively, then

(1) $2m_A = m_B$
(2) $3m_A = m_B$
(3) $4m_A = m_B$
(4) $\sqrt{3}m_A = m_B$

54. A shell of mass m moving with a velocity v breakes up suddenly into two pieces. The part having mass $\frac{m}{3}$ remains stationary. The velocity of the other part will be

(1) v
(2) $2v$
(3) $\frac{2}{3}v$
(4) $\frac{3}{2}v$

55. A particle of mass m moving towards west with speed v collides with another particle of mass m moving towards south. If two particles stick to each other, the speed of the new particle of mass $2m$ will be

(1) $v\sqrt{2}$
(2) $\frac{v}{\sqrt{2}}$
(3) $\frac{v}{2}$
(4) v

56. A body of mass 10 kg moving with speed of 3 ms^{-1} collides with another stationary body of mass 5 kg. As a result, the two bodies stick together. The KE of composite mass will be

(1) 30 J
(2) 60 J
(3) 90 J
(4) 120 J

57. A stationary particle explodes into two particles of masses x and y, which move in opposite directions with velocity v_1 and v_2. The ratio of their kinetic energies $(E_1 : E_2)$ is

(1) 1
(2) $\frac{xv_2}{yv_1}$
(3) $\frac{x}{y}$
(4) $\frac{y}{x}$

58. Select the false statement

(1) In elastic collision, KE is not conserved during the collision
(2) The coefficient of restitution for a collision between two steel balls lies between 0 and 1
(3) The momentum of a ball colliding elastically with the floor is conserved
(4) In an oblique elastic collision between two identical bodies with one of them at rest initially, the final velocities are perpendicular

59. A bullet of mass m moving with a velocity u strikes a block of mass M at rest and gets embedded in the block. The loss of kinetic energy in the impact is

(1) $\frac{1}{2}mMu^2$
(2) $\frac{1}{2}(m+M)u^2$
(3) $\frac{mMu^2}{2(m+M)}$
(4) $\left(\frac{m+M}{2mM}\right)u^2$

60. A bullet of mass m moving with velocity v strikes a suspended wooden block of mass M. If the block rises to height h, the initial velocity of the bullet will be

(1) $\sqrt{2gh}$
(2) $\frac{M+m}{m}\sqrt{2gh}$
(3) $\frac{m}{M+m}\sqrt{2gh}$
(4) $\frac{M+m}{M}\sqrt{2gh}$

61. A ball is allowed to fall from a height of 10 m. If there is 40% loss of energy due to impact, then after one impact ball will go up by

(1) 10 m
(2) 8 m
(3) 4 m
(4) 6 m

62. A bullet weighing 10 g and moving with a velocity 300 m/s strikes a 5 kg block of ice and drop dead. The ice block is kept on smooth surface. The speed of the block after the collision is

(1) 6 cm/s
(2) 60 cm/s
(3) 6 m/s
(4) 0.6 cm/s

63. A particle of mass m moving eastward with a speed v collides with another particle of the same mass moving northwards with same speed v. The two particles coalesce on collision. The new particle of mass $2m$ will move with velocity

(1) $\frac{v}{2}$ North-East
(2) $\frac{v}{\sqrt{2}}$ South-West
(3) $\frac{v}{2}$ North-West
(4) $\frac{v}{\sqrt{2}}$ North-East

64. Two perfectly elastic particles A and B of equal masses travelling along the line joining them with velocity 15 m/s and 10 m/s respectively, collide. Their velocities after the elastic collision will be (in m/s), respectively

(1) 0, 25
(2) 3, 20
(3) 10, 15
(4) 20, 5

65. Two balls of equal mass undergo head on collision while each was moving with speed 6 m/s. If the coefficient of restitution is $\frac{1}{3}$, the speed of each ball after impact will be

(1) 18 m/s (2) 2 m/s
(3) 6 m/s (4) 4 m/s

66. Select the false statement

(1) In elastic collision, kinetic energy during the collision is not conserved

(2) The coefficient of restitution for a collision between two steel balls lies between zero and one

(3) The momentum of a ball colliding elastically with the floor is conserved

(4) In an oblique elastic collision between two identical bodies with initially one of them at rest, final velocities are perpendicular

67. A ball of mass M moving with speed v collides perfectly inelastically with another ball of mass m at rest. The magnitude of impulse imparted to the first ball is

(1) Mv

(2) mv

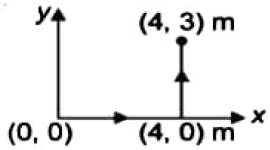
(3) $\frac{Mm}{M+m}v$

(4) $\frac{M^2}{M+m}v$

SECTION - B

Objective Type Questions

1. A force $\vec{F} = (3\vec{i} + 4\vec{j})$N acts on a particle moving in x-y plane. Starting from origin, the particle first goes along x-axis to the point (4, 0)m and then parallel to the y-axis to the point (4, 3)m. The total work done by the force on the particle is



(1) + 12 J (2) – 6 J
(3) + 24 J (4) – 12 J

2. A body of mass m is allowed to fall with the help of string with downward acceleration $\frac{g}{6}$ to a distance x. The work done by the string is

(1) $\frac{mgx}{6}$ (2) $-\frac{mgx}{6}$
(3) $\frac{5mgx}{6}$ (4) $-\frac{5mgx}{6}$

3. A chain is on a frictionless table with one fifth of its length hanging over the edge. If the chain has length L and mass M, the work required to be done to pull the hanging part back onto the table is

(1) $\frac{MgL}{5}$ (2) $\frac{MgL}{50}$
(3) $\frac{MgL}{18}$ (4) $\frac{MgL}{10}$

4. A bullet of mass 20 g leaves a riffle at an initial speed 100 m/s and strikes a target at the same level with speed 50 m/s. The amount of work done by the resistance of air will be

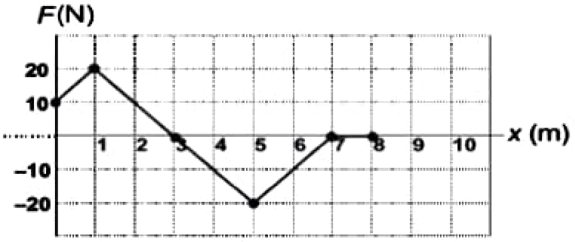
(1) 100 J (2) 25 J
(3) 75 J (4) 50 J

5. A stone with weight w is thrown vertically upward into the air from ground level with initial speed v_0. If a constant force f due to air drag acts on the stone throughout its flight. The maximum height attained by the stone is

(1) $h = \frac{v_0^2}{2g\left(1+\frac{f}{w}\right)}$ (2) $h = \frac{v_0^2}{2g\left(1-\frac{f}{w}\right)}$

(3) $h = \frac{v_0^2}{2g\left(1+\frac{w}{f}\right)}$ (4) $h = \frac{v_0^2}{2g\left(1-\frac{w}{f}\right)}$

6. Figure shows the variation of a force F acting on a particle along x-axis. If the particle begins at rest at $x = 0$, what is the particle's coordinate when it again has zero speed?



(1) $x = 3$ (2) $x = 6$
(3) $x = 5$ (4) $x = 7$

7. A spring of force constant K is first stretched by distance a from its natural length and then further by distance b. The work done in stretching the part b is

(1) $\frac{1}{2}Ka(a-b)$ (2) $\frac{1}{2}Ka(a+b)$

(3) $\frac{1}{2}Kb(a-b)$ (4) $\frac{1}{2}Kb(2a+b)$

8. A knife of mass m is at a height x from a large wooden block. The knife is allowed to fall freely, strikes the block and comes to rest after penetrating distance y. The work done by the wooden block to stop the knife is

(1) mgx (2) $-mgy$

(3) $-mg(x+y)$ (4) $mg(x-y)$

9. A man is running on horizontal road has half the kinetic energy of a boy of half of his mass. When man speeds up by 1 m/s, then his KE becomes equal to KE of the boy, the original speed of the man is

(1) $\sqrt{2}$ m/s (2) $\left(\sqrt{2}-1\right)$ m/s

(3) 2 m/s (4) $\left(\sqrt{2}+1\right)$ m/s

10. A particle of mass m starts moving from origin along x-axis and its velocity varies with position (x) as $v = k\sqrt{x}$. The work done by force acting on it during first "t" seconds is

(1) $\frac{mk^4t^2}{4}$ (2) $\frac{mk^2t}{2}$

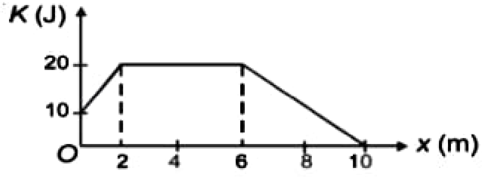
(3) $\frac{mk^4t^2}{8}$ (4) $\frac{mk^2t^2}{4}$

11. A particle is moving in a circular path of radius r under the action of a force F. If at an instant velocity of particle is v, and speed of particle is increasing, then

(1) $\vec{F}.\vec{v}=0$ (2) $\vec{F}.\vec{v}>0$

(3) $\vec{F}.\vec{v}<0$ (4) $\vec{F}.\vec{v}\geq 0$

12. The kinetic energy K of a particle moving along x-axis varies with its position (x) as shown in figure

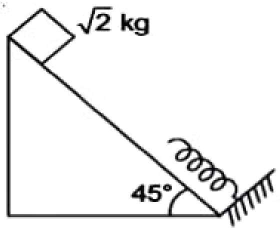


The magnitude of force acting on particle at $x = 9$ m is

(1) Zero (2) 5 N

(3) 20 N (4) 7.5 N

13. A block of mass $\sqrt{2}$ kg is released from the top of an inclined smooth surface as shown in figure. If spring constant of spring is 100 N/m and block comes to rest after compressing the spring by 1 m, then the distance travelled by block before it comes to rest is



(1) 1 m (2) 1.25 m

(3) 2.5 m (4) 5 m

14. If net force on a system is zero then

(1) Its momentum is conserved

(2) Its kinetic energy may increase

(3) The acceleration of its a constituent particle may be non-zero

(4) All of these

15. Internal forces acting within a system of particles can alter

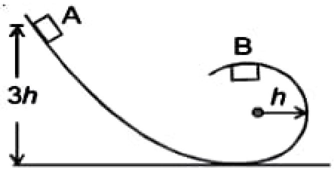
(1) The linear momentum as well as the kinetic energy of the system

(2) The linear momentum of the system, but not the kinetic energy of the system

(3) The kinetic energy of the system, but not the linear momentum of the system

(4) Neither linear momentum nor kinetic energy of the system

16. In the figure shown, a particle is released from the position A on a smooth track. When the particle reaches at B, then normal reaction on it by the track is



(1) mg (2) $2mg$

(3) $\frac{2}{3}mg$ (4) $\frac{m^2g}{h}$

17. A particle of mass m is projected with speed u at angle θ with horizontal from ground. The work done by gravity on it during its upward motion is

(1) $\dfrac{-mu^2 \sin^2\theta}{2}$ (2) $\dfrac{mu^2 \cos^2\theta}{2}$

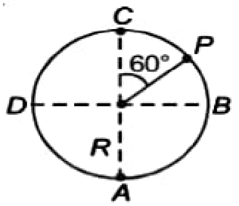
(3) $\dfrac{mu^2 \sin^2\theta}{2}$ (4) Zero

18. A shell at rest on a smooth horizontal surface explodes into two fragments of masses m_1 and m_2. If just after explosion m_1 move with speed u, then work done by internal forces during explosion is

(1) $\dfrac{1}{2}(m_1+m_2)\dfrac{m_2}{m_1}u^2$ (2) $\dfrac{1}{2}(m_1+m_2)u^2$

(3) $\dfrac{1}{2}m_1u^2\left(1+\dfrac{m_1}{m_2}\right)$ (4) $\dfrac{1}{2}(m_2-m_1)u^2$

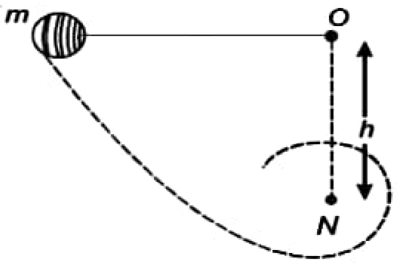
19. A particle is moving along a vertical circle of radius R. At P, what will be the velocity of particle (assume critical condition at C)?



(1) $\sqrt{gR}$ (2) $\sqrt{2gR}$

(3) $\sqrt{3gR}$ (4) $\sqrt{\dfrac{3}{2}gR}$

20. A particle of mass m attached to the end of string of length l is released from the horizontal position. The particle rotates in a circle about O as shown. When it is vertically below O, the string makes contact with a nail N placed directly below O at a distance h and rotates around it. For the particle to swing completely around the nail in a circle,



(1) $h < \dfrac{3}{5}l$ (2) $h \geq \dfrac{3}{5}l$

(3) $h < \dfrac{2}{5}l$ (4) $h \geq \dfrac{2}{5}l$

21. If $F = 2x^2 - 3x - 2$, then select the correct statement

(1) $x = -\dfrac{1}{2}$ is the position of stable equilibrium

(2) $x = 2$ is the position of stable equilibrium

(3) $x = -\dfrac{1}{2}$ is the position of unstable equilibrium

(4) $x = 2$ is the position of neutral equilibrium

22. When a conservative force does positive work on a body, then the

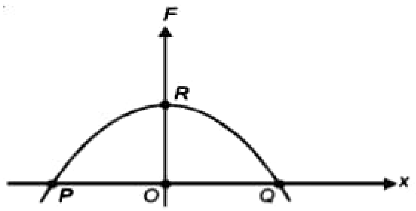
(1) Potential energy of body increases

(2) Potential energy of body decreases

(3) Total mechanical energy of body increases

(4) Total mechanical energy of body decreases

23. The variation of force F acting on a body moving along x-axis varies with its position (x) as shown in figure



The body is in stable equilibrium state at

(1) P (2) Q

(3) R (4) Both P & Q

24. A particle located in one dimensional potential field has potential energy function $U(x) = \dfrac{a}{x^2} - \dfrac{b}{x^3}$, where a and b are positive constants. The position of equilibrium corresponds to $x =$

(1) $\dfrac{3a}{2b}$ (2) $\dfrac{2b}{3a}$

(3) $\dfrac{2a}{3b}$ (4) $\dfrac{3b}{2a}$

25. The force required to row a boat at constant velocity is proportional to square of its speed. If a speed of v km/h requires 4 kW, how much power does a speed of $2v$ km/h require?

(1) 8 kW (2) 16 kW

(3) 24 kW (4) 32 kW

26. A body of mass *m* is projected from ground with speed *u* at an angle θ with horizontal. The power delivered by gravity to it at half of maximum height from ground is

(1) $\frac{mgu\cos\theta}{\sqrt{2}}$ (2) $\frac{mgu\sin\theta}{\sqrt{2}}$

(3) $\frac{mgu\cos(90+\theta)}{\sqrt{2}}$ (4) Both (2) & (3)

27. A particle of mass *m* moves in a circular path of radius *r*, under the action of force which delivers it constant power *p* and increases its speed. The angular acceleration of particle at time (*t*) is proportional

(1) $\frac{1}{\sqrt{t}}$ (2) $\sqrt{t}$

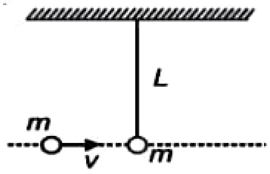
(3) t^0 (4) $t^{3/2}$

28. The rate of doing work by force acting on a particle moving along *x*-axis depends on position *x* of particle and is equal to 2*x*. The velocity of particle is given by expression

(1) $\left[\frac{3x^2}{m}\right]^{1/3}$ (2) $\left[\frac{3x^2}{2m}\right]^{1/3}$

(3) $\left(\frac{2mx}{9}\right)^{1/2}$ (4) $\left[\frac{mx^2}{3}\right]^{1/2}$

29. A small ball of mass *m* moving with speed *v* ($< \sqrt{2gL}$) undergoes an elastic head on collision with a stationary bob of identical mass of a simple pendulum of length *L*. The maximum height *h*, from the equilibrium position, to which the bob rises after collision is



(1) $\frac{v^2}{2g}$ (2) $\frac{v^2}{4g}$

(3) $\frac{v^2}{8g}$ (4) $\frac{3v^2}{8g}$

30. Two balls of masses *m* each are moving at right angle to each other with velocities 6 m/s and 8 m/s respectively. If collision between them is perfectly inelastic, the velocity of combined mass is

(1) 15 m/s (2) 10 m/s

(3) 5 m/s (4) 2.5 m/s

31. A sphere of mass *m* moving with a constant velocity *u* hits another stationary sphere of the same mass. If *e* is the coefficient of restitution, then ratio of velocities of the two spheres after collision will be

(1) $\frac{1-e}{1+e}$ (2) $\frac{2+e}{2-e}$

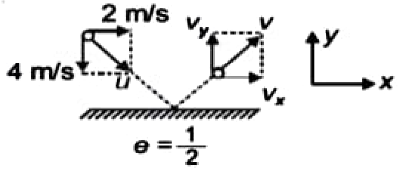
(3) $\left(\frac{1+e}{1-e}\right)^2$ (4) $\left(\frac{1-e}{1+e}\right)^2$

32. A neutron travelling with a velocity collides elastically, head on, with a nucleus of an atom of mass number *A* at rest. The fraction of total energy retained by neutron is

(1) $\left(\frac{A-1}{A+1}\right)^2$ (2) $\left(\frac{A+1}{A-1}\right)^2$

(3) $\left(\frac{A-1}{A}\right)^2$ (4) $\left(\frac{A+1}{A}\right)^2$

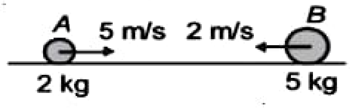
33. In the figure shown, a small ball hits obliquely a smooth and horizontal surface with speed *u* whose *x* and *y* components are indicated. If the coefficient of restitution is $\frac{1}{2}$, then its *x* and *y* components v_x and v_y just after collision are respectively



(1) 4 m/s, 1 m/s (2) 2 m/s, 1 m/s

(3) 2 m/s, 2 m/s (4) 4 m/s, 2 m/s

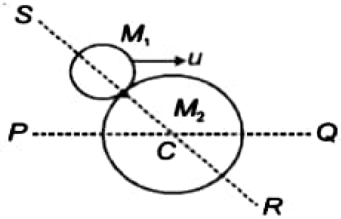
34. Velocity of the ball A after collision with the ball B as shown in the figure is (Assume perfectly inelastic and head-on collision)



(1) $\frac{3}{7}$ m/s (2) $\frac{5}{7}$ m/s

(3) $\frac{1}{7}$ m/s (4) Zero

35. An object of mass M_1 moving horizontally with speed u collides elastically with another object of mass M_2 at rest. Select correct statement.



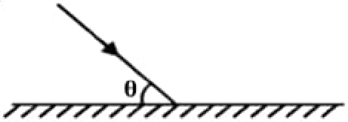
(1) The momentum of system is conserved only in direction *PQ*

(2) Momentum of M_1 is conserved in direction perpendicular to *SR*

(3) Momentum of M_2 will change in direction normal to *CR*

(4) All of these

36. A ball of mass m moving with speed u collides with a smooth horizontal surface at angle θ with it as shown in figure. The magnitude of impulse imparted to surface by ball is [Coefficient of restitution of collision is e]



(1) $mu(1 + e)\cos\theta$ (2) $mu(1 - e)\sin\theta$

(3) $mu(1 - e)\cos\theta$ (4) $mu(1 + e)\sin\theta$

37. A body of mass m falls from height h on ground. If e be the coefficient of restitution of collision between the body and ground, then the distance travelled by body before it comes to rest is

(1) $h\left\{\frac{1+e^2}{1-e^2}\right\}$ (2) $h\left\{\frac{1-e^2}{1+e^2}\right\}$

(3) $\frac{2eh}{1+e^2}$ (4) $\frac{2eh}{1-e^2}$

38. The PE of a 2 kg particle, free to move along x-axis is given by $V(x) = \left(\frac{x^3}{3} - \frac{x^2}{2}\right)$J. The total mechanical energy of the particle is 4 J. Maximum speed (in ms^{-1}) is

(1) $\frac{1}{\sqrt{2}}$ (2) $\sqrt{2}$

(3) $\frac{3}{\sqrt{2}}$ (4) $\frac{5}{\sqrt{6}}$

39. A bullet of mass m moving with velocity v strikes a suspended wooden block of mass M. If the block rises to height h, then the initial velocity v of the bullet must have been

(1) $\sqrt{2gh}$ (2) $\frac{M+m}{m}\sqrt{2gh}$

(3) $\frac{m}{M+m}\sqrt{2gh}$ (4) $\frac{M+m}{M}\sqrt{2gh}$

SECTION - C

Previous Years Questions

1. A particle moves from a point $(-2\hat{i}+5\hat{j})$ to $(4\hat{j}+3\hat{k})$ when a force of $(4\hat{i}+3\hat{j})$N is applied. How much work has been done by the force? **[NEET (Phase-2) 2016]**

(1) 8 J (2) 11 J

(3) 5 J (4) 2 J

2. A bullet of mass 10 g moving horizontally with a velocity of 400 ms^{-1} strikes a wood block of mass 2 kg which is suspended by light inextensible string of length 5 m. As a result, the centre of gravity of the block found to rise a vertical distance of 10 cm. The speed of the bullet after it emerges out horizontally from the block will be **[NEET (Phase-2) 2016]**

(1) 100 ms^{-1} (2) 80 ms^{-1}

(3) 120 ms^{-1} (4) 160 ms^{-1}

3. Two identical balls *A* and *B* having velocities of 0.5 m/s and –0.3 m/s respectively collide elastically in one dimension. The velocities of *B* and *A* after the collision respectively will be **[NEET (Phase-2) 2016]**

(1) –0.5 m/s and 0.3 m/s

(2) 0.5 m/s and –0.3 m/s

(3) –0.3 m/s and 0.5 m/s

(4) 0.3 m/s and 0.5 m/s

4. What is the minimum velocity with which a body of mass m must enter a vertical loop of radius R so that it can complete the loop? **[NEET-2016]**

(1) $\sqrt{5gR}$ (2) $\sqrt{gR}$

(3) $\sqrt{2gR}$ (4) $\sqrt{3gR}$

5. A body of mass 1 kg begins to move under the action of a time dependent force $F = (2t\hat{i} + 3t^2\hat{j})$N, where $\hat{i}$ and $\hat{j}$ are unit vectors along x and y axis. What power will be developed by the force at the time t? **[NEET-2016]**

(1) $(2t^3 + 3t^5)$ W (2) $(2t^2 + 3t^2)$ W

(3) $(2t^2 + 4t^4)$ W (4) $(2t^3 + 3t^4)$ W

6. If vectors $\vec{A} = \cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}$ and $\vec{B} = \cos\frac{\omega t}{2}\hat{i} + \sin\frac{\omega t}{2}\hat{j}$ are functions of time, then the value of t at which they are orthogonal to each other is **[Re-AIPMT-2015]**

(1) $t = 0$ (2) $t = \frac{\pi}{4\omega}$

(3) $t = \frac{\pi}{2\omega}$ (4) $t = \frac{\pi}{\omega}$

7. A ball is thrown vertically downwards from a height of 20 m with an initial velocity v_0^2. It collides with the ground, loses 50 percent of its energy in collision and rebounds to the same height. The initial velocity v_0 is (Take g = 10 ms^{-2}) **[Re-AIPMT-2015]**

(1) 10 ms^{-1} (2) 14 ms^{-1}

(3) 20 ms^{-1} (4) 28 ms^{-1}

8. On a frictionless surface, a block of mass M moving at speed v collides elastically with another block of same mass M which is initially at rest. After collision the first block moves at an angle θ to its initial direction and has a speed $\frac{v}{3}$. The second blocks speed after the collision is **[Re-AIPMT-2015]**

(1) $\frac{\sqrt{3}}{2}v$ (2) $\frac{2\sqrt{2}}{3}v$

(3) $\frac{3}{4}v$ (4) $\frac{3}{\sqrt{2}}v$

9. The heart of a man pumps 5 litres of blood through the arteries per minute at a pressure of 150 mm of mercury. If the density of mercury be 13.6 × 10^3 kg/m^3 and g = 10 m/s^2 then the power of heart in watt is **[Re-AIPMT-2015]**

(1) 1.50 (2) 1.70

(3) 2.35 (4) 3.0

10. Two particles of masses m_1, m_2 move with initial velocities u_1 and u_2. On collision, one of the particles get excited to higher level, after absorbing energy ε. If final velocities of particles be v_1 and v_2 then we must have **[AIPMT-2015]**

(1) $\frac{1}{2}m_1^2u_1^2 + \frac{1}{2}m_2^2u_2^2 + \varepsilon = \frac{1}{2}m_1^2v_1^2 + \frac{1}{2}m_2^2v_2^2$

(2) $m_1^2u_1 + m_2^2u_2 - \varepsilon = m_1^2v_1 + m_2^2v_2$

(3) $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \varepsilon$

(4) $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \varepsilon = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

11. Two similar springs P and Q have spring constants K_P and K_Q such that $K_P > K_Q$. They stretched first by the same amount (case a), then by the same force (case b). The work done by the springs W_P and W_Q are related as in case (a) and case (b), respectively **[AIPMT-2015]**

(1) $W_P < W_Q$; $W_Q < W_P$

(2) $W_P = W_Q$; $W_P > W_Q$

(3) $W_P = W_Q$; $W_P = W_Q$

(4) $W_P > W_Q$; $W_Q > W_P$

12. A particle of mass m is driven by a machine that delivers a constant power k watts. If the particle starts from rest+ the force on the particle at time t is **[AIPMT-2015]**

(1) $\frac{1}{2}\sqrt{mk}\,t^{\frac{-1}{2}}$ (2) $\sqrt{\frac{mk}{2}}\,t^{\frac{-1}{2}}$

(3) $\sqrt{mk}\,t^{\frac{-1}{2}}$ (4) $\sqrt{2mk}\,t^{\frac{-1}{2}}$

13. A block of mass 10 kg moving in x direction with a constant speed of 10 ms^{-1}, is subjected to a retarding force $F = 0.1x$ J/m during its travel from x = 20 m to 30 m. Its final KE will be **[AIPMT-2015]**

(1) 250 J (2) 475 J

(3) 450 J (4) 275 J

14. A body of mass (4m) is lying in x-y plane at rest. It suddenly explodes into three pieces. Two pieces, each of mass (m) move perpendicular to each other with equal speeds (v). The total kinetic energy generated due to explosion is **[AIPMT-2014]**

(1) mv^2 (2) $\frac{3}{2}mv^2$

(3) $2mv^2$ (4) $4mv^2$

15. A uniform force of $(3\hat{i}+\hat{j})$ newton acts on a particle of mass 2 kg. Hence the particle is displaced from position $(2\hat{i}+\hat{k})$ metre to position $(4\hat{i}+3\hat{j}-\hat{k})$ metre. The work done by the force on the particle is **[NEET-2013]**

(1) 6 J (2) 13 J

(3) 15 J (4) 9 J

16. The potential energy of a particle in a force field is $U=\frac{A}{r^2}-\frac{B}{r}$, where A and B are positive constants and r is the distance of particle from the centre of the field. For stable equilibrium, the distance of the particle is **[AIPMT (Prelims)-2012]**

(1) $\frac{A}{B}$ (2) $\frac{B}{A}$

(3) $\frac{B}{2A}$ (4) $\frac{2A}{B}$

17. Two spheres A and B of masses m_1 and m_2 respectively collide. A is at rest initially and B is moving with velocity v along x-axis. After collision B has a velocity $\frac{v}{2}$ in a direction perpendicular to the original direction. The mass A moves after collision in the direction **[AIPMT (Prelims)-2012]**

(1) $\theta = \tan^{-1}\left(\frac{1}{2}\right)$ to the x-axis

(2) $\theta = \tan^{-1}\left(-\frac{1}{2}\right)$ to the y-axis

(3) Same as that of B

(4) Opposite to that of B

18. A stone is dropped from a height h. It hits the ground with a certain momentum P. If the same stone is dropped from a height 100% more than the previous height, the momentum when it hits the ground will change by **[AIPMT (Mains)-2012]**

(1) 68% (2) 41%

(3) 200% (4) 100%

19. A car of mass m starts from rest and accelerates so that the instantaneous power delivered to the car has a constant magnitude P_0. The instantaneous velocity of this car is proportional to

[AIPMT (Mains)-2012]

(1) t^2P_0 (2) $t^{1/2}$

(3) $t^{-1/2}$ (4) $\frac{1}{\sqrt{m}}$

20. The potential energy of a system increases if work is done **[AIPMT (Prelims)-2011]**

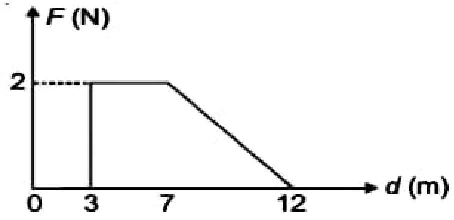
(1) Upon the system by a conservative force

(2) Upon the system by a nonconservative force

(3) By the system against a conservative force

(4) By the system against a non conservative force

21. Force F on a particle moving in a straight line varies with distance d as shown in the figure. The work done on the particle during its displacement of 12 m is **[AIPMT (Prelims)-2011]**



(1) 13 J (2) 18 J

(3) 21 J (4) 26 J

22. A body projected vertically form the earth reaches a height equal to earth's radius before returning to the earth. The power exerted by the gravitational force is greatest **[AIPMT (Prelims)-2011]**

(1) At the instant just after the body is projected

(2) At the highest position of the body

(3) At the instant just before the body hits the earth

(4) It remains constant all through

23. An engine pumps water through a hose pipe. Water passes through the pipe and leaves it with a velocity of 2 m/s. The mass per unit length of water in the pipe is 100 kg/m. What is the power of the engine?

[AIPMT (Prelims)-2010]

(1) 300 W (2) 400 W

(3) 200 W (4) 100 W

24. A particle of mass M starting from rest undergoes uniform acceleration. If the speed acquired in time T is V, the power delivered to the particle is

[AIPMT (Mains)-2010]

(1) $\frac{MV^2}{T}$ (2) $\frac{1}{2}\frac{MV^2}{T^2}$

(3) $\frac{MV^2}{T^2}$ (4) $\frac{1}{2}\frac{MV^2}{T}$

25. An engine pumps water continuously through a hose. Water leaves the hose with a velocity v and m is the mass per unit length of the water jet. What is the rate at which kinetic energy is imparted water?

[AIPMT (Prelims)-2009]

(1) mv^2
(2) $\frac{1}{2}mv^2$
(3) $\frac{1}{2}m^2v^2$
(4) $\frac{1}{2}mv^3$

26. A body of mass 1 kg is thrown upwards with a velocity 20 m/s. It momentarily comes to rest after attaining a height of 18 m. How much energy is lost due to air friction? (g = 10 m/s^2)

[AIPMT (Prelims)-2009]

(1) 30 J
(2) 40 J
(3) 10 J
(4) 20 J

27. A block of mass M is attached to the lower end of a vertical spring. The spring is hung from a ceiling and has force constant value k. The mass is released from rest with the spring initially unstretched. The maximum extension produced in the length of the spirng will be: **[AIPMT (Prelims)-2009]**

(1) $\frac{2Mg}{k}$
(2) $\frac{4Mg}{k}$
(3) $\frac{Mg}{2k}$
(4) $\frac{Mg}{k}$

28. Water falls from a height of 60 m at the rate of 15 kg/s to operate a turbine. The losses due to frictional forces are 10% of energy. How much power is generated by the turbine? (g = 10 m/s^2)

[AIPMT (Prelims)-2008]

(1) 7.0 kW
(2) 8.1 kW
(3) 10.2 kW
(4) 12.3 kW

29. A shell of mass 200 gm is ejected from a gun of mass 4 kg by an explosion that generates 1.05 kJ of energy. The initial velocity of the shell is

[AIPMT (Prelims)-2008]

(1) 120 ms^{-1}
(2) 100 ms^{-1}
(3) 80 ms^{-1}
(4) 40 ms^{-1}

30. A vertical spring with force constant K is fixed on a table. A ball of mass m at a height h above the free upper end of the spring falls vertically on the spring so that the spring is compressed by a distance d. The net work done in the process is

[AIPMT (Prelims)-2007]

(1) $mg(h - d) + \frac{1}{2}Kd^2$
(2) $mg(h + d) + \frac{1}{2}Kd^2$
(3) $mg(h + d) - \frac{1}{2}Kd^2$
(4) $mg(h - d) - \frac{1}{2}Kd^2$

31. The potential energy of a long spring when stretched by 2 cm is U. If the spring is stretched by 8 cm the potential energy stored in it is :

[AIPMT (Prelims)-2006]

(1) $4U$
(2) $8U$
(3) $16U$
(4) $\frac{U}{4}$

32. A body of mass 3 kg is under a constant force which causes a displacement s in metres in it, given by the relation $s = \frac{1}{3}t^2$, where t is in s. Workdone by the force in 2 s is

[AIPMT (Prelims)-2006]

(1) $\frac{5}{19}$ J
(2) $\frac{3}{8}$ J
(3) $\frac{8}{3}$ J
(4) $\frac{19}{5}$ J

33. 300 J of work is done in sliding a 2 kg block up an inclined plane of height 10 m. Taking g = 10 m/s^2, work done against friction is

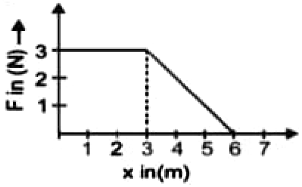
[AIPMT (Prelims)-2006]

(1) 200 J
(2) 100 J
(3) Zero
(4) 1000 J

34. A bomb of mass 30 kg at rest explodes into two pieces of masses 18 kg and 12 kg. The velocity of 18 kg mass is 6 ms^{-1}. The kinetic energy of the other mass is **[AIPMT (Prelims)-2005]**

(1) 256 J
(2) 486 J
(3) 524 J
(4) 324 J

35. A force F acting on an object varies with distance x as shown here. The force is in N and x in m. The work done by the force in moving the object from $x = 0$ to $x = 6$ m is **[AIPMT (Prelims)-2005]**



(1) 4.5 J

(2) 13.5 J

(3) 9.0 J

(4) 18.0 J

Questions asked Prior to Medical Ent. Exams. 2005

36. The angle between the two vectors $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ will be

(1) 90°

(2) 180°

(3) Zero

(4) 45°

37. Vectors $\vec{A}, \vec{B}$ and $\vec{C}$ are such that $\vec{A} \cdot \vec{B} = 0$ and $\vec{A} \cdot \vec{C} = 0$. Then the vector parallel to $\vec{A}$ is

(1) $\vec{B}$ and $\vec{C}$

(2) $\vec{A} \times \vec{B}$

(3) $\vec{B} + \vec{C}$

(4) $\vec{B} \times \vec{C}$

38. If a unit vector is represented by $0.5\hat{i} - 0.8\hat{j} + c\hat{k}$ then the value of c is

(1) $\sqrt{0.01}$

(2) $\sqrt{0.11}$

(3) 1

(4) $\sqrt{0.39}$

39. If a vector $2\hat{i} + 3\hat{j} + 8\hat{k}$ is perpendicular to the vector $4\hat{j} - 4\hat{i} + \alpha\hat{k}$, then the value of α is

(1) $\frac{1}{2}$

(2) $-\frac{1}{2}$

(3) 1

(4) –1

40. The work done by an applied variable force $F = x + x^3$ from $x = 0$ m to $x = 2$ m, where x is displacement, is

(1) 6 J

(2) 8 J

(3) 10 J

(4) 12 J

41. When a body moves with a constant speed along a circle

(1) No work is done on it

(2) No acceleration is produced in it

(3) Its velocity remains constant

(4) No force acts on it

42. A position dependent force, $F = (7 - 2x + 3x^2)$ N acts on a small body of mass 2 kg and displaces it from $x = 0$ to $x = 5$ m. The work done in joules is

(1) 135

(2) 270

(3) 35

(4) 70

43. A body, constrained to move in y-direction, is subjected to a force given by $\vec{F} = \left(-2\hat{i} + 15\hat{j} + 6\hat{k}\right)$N. The work done by this force in moving the body through a distance of 10 m along positive y-axis, is

(1) 150 J

(2) 20 J

(3) 190 J

(4) 160 J

44. A body moves a distance of 10 m along a straight line under the action of a 5 N force. If the work done is 25 J, then angle between the force and direction of motion of the body is

(1) 60°

(2) 75°

(3) 30°

(4) 45°

45. A force acts on a 3 g particle in such a way that the position of the particle as a function of time is given by $x = 3t - 4t^2 + t^3$, where x is in metres and t is in seconds. The work done during the first 4 second is

(1) 490 mJ

(2) 450 mJ

(3) 576 mJ

(4) 528 mJ

46. Two bodies of masses m and $4m$ are moving with equal K.E. The ratio of their linear momenta is

(1) 1 : 2

(2) 1 : 4

(3) 4 : 1

(4) 1 : 1

47. One kilowatt hour is equal to

(1) 36×10^{-5} J

(2) 36×10^{5} J

(3) 36×10^{7} J

(4) 36×10^{3} J

48. Two bodies with kinetic energies in the ratio of 4 : 1 are moving with equal linear momentum. The ratio of their masses is

(1) 4 : 1 (2) 1 : 1

(3) 1 : 2 (4) 1 : 4

49. A 1 kg stationary bomb is exploded in three parts having masses in ratio 1 : 1 : 3 respectively. Parts having same mass move in perpendicular direction with velocity 30 m/s, then the velocity of bigger part will be

(1) $10\sqrt{2}$ m/s (2) $\frac{10}{\sqrt{2}}$ m/s

(3) $15\sqrt{2}$ m/s (4) $\frac{15}{\sqrt{2}}$ m/s

50. If kinetic energy of a body is increased by 300% then percentage change in momentum will be

(1) 100% (2) 150%

(3) 265% (4) 73.2%

51. A stationary particle explodes into two particles of masses m_1 and m_2 which move in opposite directions with velocities v_1 and v_2. The ratio of their kinetic energies $\frac{E_1}{E_2}$ is

(1) $\frac{m_2}{m_1}$ (2) $\frac{m_1}{m_2}$

(3) 1 (4) $\frac{m_1 v_2}{m_2 v_1}$

52. A particle of mass m_1 is moving with a velocity v_1 and another particle of mass m_2 is moving with a velocity v_2. Both of them have the same momentum but their different kinetic energies are E_1 and E_2 respectively. If $m_1 > m_2$, then

(1) $E_1 < E_2$ (2) $\frac{E_1}{E_2} = \frac{m_1}{m_2}$

(3) $E_1 > E_2$ (4) $E_1 = E_2$

53. A bomb of mass 30 kg at rest explodes into two pieces of masses 18 kg and 12 kg. The velocity of 18 kg mass is 6 ms^{-1}. The kinetic energy of the other mass is

(1) 324 J (2) 486 J

(3) 256 J (4) 524 J

54. A ball whose kinetic energy is E is thrown at an angle of 45° with the horizontal. Its K.E. at the highest point of its flight will be

(1) $\frac{E}{\sqrt{2}}$ (2) Zero

(3) E (4) $\frac{E}{2}$

55. A body dropped from a height h with initial velocity zero, strikes the ground with a velocity 3 m/s. Another body of same mass is thrown from the same height h with an initial velocity of 4 m/s. Find the final velocity of second mass, with which it strikes the ground.

(1) 5 m/s (2) 12 m/s

(3) 3 m/s (4) 4 m/s

56. A particle with total energy E is moving in a potential energy region $U(x)$. Motion of the particle is restricted to the region when

(1) $U(x) > E$ (2) $U(x) < E$

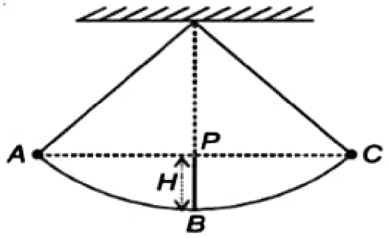
(3) $U(x) = 0$ (4) $U(x) \leq E$

57. The kinetic energy acquired by a mass m in travelling distance d, starting from rest, under the action of a constant force is directly proportional to

(1) m (2) m^0

(3) $\sqrt{m}$ (4) $\frac{1}{\sqrt{m}}$

58. A simple pendulum with a bob of mass m oscillates from A to C and back to A such that PB is H. If the acceleration due to gravity is g, then the velocity of the bob as it passes through B is



(1) mgH

(2) $\sqrt{2gH}$

(3) Zero

(4) $2gH$

59. A car moving with a speed of 40 km/h can be stopped by applying brakes after at least 2 m. If the same car is moving with a speed of 80 km/h, what is the minimum stopping distance?

(1) 4 m (2) 6 m

(3) 8 m (4) 2 m

60. A child is sitting on a swing. Its minimum and maximum heights from the ground are 0.75 m and 2 m respectively, its maximum speed will be

(1) 10 m/s (2) 5 m/s

(3) 8 m/s (4) 15 m/s

61. When a long spring is stretched by 2 cm, its potential energy is U. If the spring is stretched by 10 cm, the potential energy stored in it will

(1) $\frac{U}{5}$ (2) $5U$

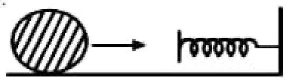
(3) $10U$ (4) $25U$

62. A ball of mass 2 kg and another of mass 4 kg are dropped together from a 60 feet tall building. After a fall of 30 feet each towards earth, their respective kinetic energies will be in the ratio of

(1) $\sqrt{2}:1$ (2) 1 : 4

(3) 1 : 2 (4) $1:\sqrt{2}$

63. A mass of 0.5 kg moving with a speed of 1.5 m/s on a horizontal smooth surface, collides with a nearly weightless spring of force constant $k = 50$ N/m. The maximum compression of the spring would be



(1) 0.15 m (2) 0.12 m

(3) 1.5 m (4) 0.5 m

64. One coolie takes 1 minute to raise a suitcase through a height of 2 m but the second coolie takes 30 s to raise the same suitcase to the same height. The powers of two coolies are in the ratio

(1) 1 : 2 (2) 1 : 3

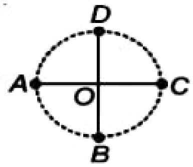
(3) 2 : 1 (4) 3 : 1

65. If a force of 9 N is acting on a body, then find instantaneous power supplied to the body when its velocity is 5 m/s in the direction of force

(1) 195 watt (2) 45 watt

(3) 75 watt (4) 100 watt

66. As shown in figure, a particle of mass m is performing vertical circular motion. The velocity of the particle is increased, then at which point will the string break?



(1) A (2) B

(3) C (4) D

67. The bob of simple pendulum having length l, is displaced from mean position to an angular position θ with respect to vertical. If it is released, then velocity of bob at equilibrium position

(1) $\sqrt{2gl(1-\cos\theta)}$ (2) $\sqrt{2gl(1+\cos\theta)}$

(3) $\sqrt{2gl\cos\theta}$ (4) $\sqrt{2gl}$

68. A stone is tied to a string of length 'l' and is whirled in a vertical circle with the other end of the string as the centre. At a certain instant of time, the stone is at its lowest position and has a speed 'u'. The magnitude of the change in velocity as it reaches a position where the string is horizontal (g being acceleration due to gravity) is

(1) $\sqrt{2(u^2-gl)}$ (2) $\sqrt{u^2-gl}$

(3) $u-\sqrt{u^2-2gl}$ (4) $\sqrt{2gl}$

69. The potential energy between two atoms, in a molecule, is given by $U(x)=\frac{a}{x^{12}}-\frac{b}{x^6}$; where a and b are positive constants and x is the distance between the atoms. The atom is in stable equilibrium, when

(1) $x=\left(\frac{2a}{b}\right)^{1/6}$

(2) $x=\left(\frac{11a}{5b}\right)^{1/6}$

(3) $x=0$

(4) $x=\left(\frac{a}{2b}\right)^{1/6}$

70. The coefficient of restitution, *e*, for a perfectly elastic collision is

(1) 0 (2) –1

(3) 1 (4) ∞

71. A particle of mass m_1 moves with velocity v_1 and collides with another particle at rest of equal mass. The velocity of the second particle after the elastic collision is

(1) $2v_1$ (2) v_1

(3) $-v_1$ (4) 0

72. Two identical balls *A* and *B* collide head-on elastically. If velocities of *A* and *B*, before the collision, are + 0.5 m/s and - 0.3 m/s respectively then their velocities, after the collision, are respectively

(1) – 0.5 m/s and + 0.3 m/s

(2) + 0.5 m/s and + 0.3 m/s

(3) + 0.3 m/s and – 0.5 m/s

(4) – 0.3 m/s and + 0.5 m/s

73. A moving body of mass *m* and velocity 3 km/h collides with a rest body of mass 2*m* and sticks to it. Now the combined mass starts to move. What will be the combined velocity?

(1) 3 km/h

(2) 4 km/h

(3) 1 km/h

(4) 2 km/h

74. A rubber ball is dropped from a height of 5 m on a plane, where the acceleration due to gravity is not known. On bouncing, it rises to 1.8 m. The ball loses its velocity on bouncing by a factor of

(1) $\frac{3}{5}$ (2) $\frac{2}{5}$

(3) $\frac{16}{25}$ (4) $\frac{9}{25}$

75. A ball moving with velocity 2 m/s collides head on with another stationary ball of double the mass. If the coefficient of restitution is 0.5 then their velocities (in m/s) after collision will be

(1) 0, 2 (2) 0, 1

(3) 1, 1 (4) 1, 0.5

76. A metal ball of mass 2 kg moving with speed of 36 km/h has a head on collision with a stationary ball of mass 3 kg. If after collision, both the balls move together, then the loss in K.E. due to collision is

(1) 100 J

(2) 140 J

(3) 40 J

(4) 60 J

77. Two springs *A* and *B* having spring constant K_A and K_B ($K_A = 2K_B$) are stretched by applying force of equal magnitude. If energy stored in spring *A* is E_A then energy stored in *B* will be

(1) $2E_A$ (2) $\frac{E_A}{4}$

(3) $\frac{E_A}{2}$ (4) $4E_A$

SECTION - D

Assertion - Reason Type Questions

In the following questions, a statement of assertion (A) is followed by a statement of reason (R)

(1) If both Assertion & Reason are true and the reason is the correct explanation of the assertion, then mark **(1)**.

(2) If both Assertion & Reason are true but the reason is not the correct explanation of the assertion, then mark **(2)**.

(3) If Assertion is true statement but Reason is false, then mark **(3)**.

(4) If both Assertion and Reason are false statements, then mark **(4)**.

1. A : The work done by a force during round trip is always zero.

R : The average value of force in round trip is zero.

2. A : The change in kinetic energy of a particle is equal to the work done on it by the net force.

R : The work-energy theorem can be used only in conservative field.

3. A : Internal forces can change the kinetic energy but not the momentum of the system.

R : The net internal force on a system is always zero.

4. A : The potential energy can be defined only in conservative field.

   R : The value of potential energy depends on the reference level (level of zero potential energy).

5. A : When a body moves in a circle the work done by the centripetal force is always zero.

   R : Centripetal force is perpendicular to displacement at every instant.

6. A : If net force acting on a system is zero, then work done on the system may be nonzero.

   R : Internal forces acting on a system can increase its kinetic energy.

7. A : During collision between two objects, the momentum of colliding objects is conserved only in direction perpendicular to line of impact.

   R : The force on colliding objects in direction perpendicular to line of impact is zero.

8. A : The potential energy of a system increases when work is done by conservative force.

   R : Kinetic energy can change into potential energy and vice-versa.

9. A : In inelastic collision, a part of kinetic energy convert into heat energy, sound energy and light energy etc.

   R : The force of interaction in an inelastic collision is non-conservative in nature.

10. A : Energy dissipated against friction depends on the path followed.

    R : Friction force is non-conservative force.

11. A : Work done by the frictional force can't be positive.

    R : Frictional force is a conservative force.

12. A : Impulse generated on one body by another body in a perfectly elastic collision is not zero.

    R : In a perfectly elastic collision, momentum of the system is always conserved and not the momentum of the individual bodies.

13. A : Power of the gravitational force on the body in a projectile motion is zero, once during its motion.

    R : At the highest point only, the component of velocity along the gravitational force is zero.

14. A : Power delivered by the tension in the wire to a body in vertical circle is always zero.

    R : Tension in the wire is equal to the centripetal force acting on the body doing vertical circular motion.

15. A : When a man is walking on a rough road, the work done by frictional force is zero.

    R : Frictional force acts in the direction of the motion of the man in this case.

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